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SPECIAL ISSUE PAPER

Knowledge diffusion in complex networks

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SUMMARY

Modern communication networks and social networks are the main tunnels of knowledge diffusion. Knowledge diffusion in complex networks is different from the epidemic-like information spreading, because individuals are willing to learn and spread knowledge to their friends and the learning process can hardly be achieved in a few conversations. In this paper, we investigate the important issue as what topological structure is suitable for knowledge diffusion. We propose a new knowledge diffusion model, where both learning and forgetting mechanisms are considered. In this model, individuals can play imparter and learner simultaneously. Comparing knowledge diffusion on a series of complex topologies, we observe that the individuals with a large degree can quickly learn more knowledge, who are beneficial to knowledge diffusion. Our results surprisingly reveal that the networks with high degree-heterogeneity are likely to be suitable for knowledge diffusion. Our finding suggests that enhancing the degree heterogeneity of existing social networks may help to improve the performance of knowledge diffusion. This result is well confirmed by our extensive simulation results. Our model therefore provides a theoretical framework for understanding knowledge diffusion in complex topologies. Copyright © 2016 John Wiley & Sons, Ltd.

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KEY WORDS: complex networks; knowledge diffusion; small-world networks

1. INTRODUCTION

The pressing needs of knowledge diffusion [1, 2] foster the revolution of modern information technology. As is expected, the revolution, in turn, brings knowledge diffusion more tunnels, no matter whether the content of knowledge is implicit or explicit [3]. Currently, with the accelerated development of data collection and the wider availability of a variety of databases, it becomes much easier to track knowledge diffusion. The channels of knowledge diffusion has evolved from face-to-face or person-to-person interactions [4, 5] to communication networks [6, 7], on which a large number of knowledge carriers, for instance, microblog, instant messaging software, video, and email, among the others, emerge one by one. However, the processes of the knowledge diffusion are complex [8–11], which makes tracing knowledge diffusion a rather challenging task [12]. Thus, early studies on knowledge diffusion in complex networks are mainly on the base of simulations on model-generated networks [1, 13]. In the studies, authors take knowledge transfer as a form of barter. They found that, compared with regular graphs and random graphs, the steady-state level of average knowledge reaches the peak when the networks exhibit clear 'small-world' properties [14].

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In a sense of citation analysis [15], knowledge diffusion can be defined as the adaptations and applications of knowledge documented in scientific publications and patents. Because of the wider availability of a variety of databases in recent years, a number of empirical results were successively reported [12, 16–18]. Taking the studies on the h-index [19] as an example, Gao *et al.* [12] proposed a citation-based directed network model with a time dimension. The authors take citations as an indicator of knowledge diffusion. This definition enables them to illustrate the process of knowledge diffusion with empirical data, which is more convincing than the pure simulation results. Chen *et al.* [16] analyzed patent citations in the field of tissue engineering. They developed and deployed an explanatory visualization technique, which provides a relatively intuitive presentation of knowledge diffusion. Liu *et al.* [17] studied the number of Essential Science Indicators fields influenced by the publications of a research group. Batagelj [18] proposed an approach to determine strength and weight for each node and directed link, respectively. The approach can be used for large networks with millions of nodes and links.

Inspired by spreading dynamics, Bettencourta *et al.* [20] tried to model the knowledge diffusion process as a disease spreading process. Take the studies on 'Feynman diagrams' as an example [21]; at the beginning of the spreading process, most authors are in the susceptible class, with a few authors in the incubator class having been in contact with the idea and a small number of adopters manifesting it.

Knowledge can refer to a familiarity, awareness, or understanding of some information. In the studies of information spreading on complex networks [22–33], however, more attention is attracted by another type of information diffusion, rumor spreading [22–30]. Rumor spreading is considered as an infection-like information-spreading process, which was proposed by Daley and Kendall [34, 35] in 1964. In the Daley and Kendall model, individuals are divided into three classes: ignorant, spreaders, and stiflers. Transitions from the ignorant state to spreaders may result from contacts between the two classes, whereas encounters between individuals who already know the rumor may lead the spreaders in them to be stiflers. Instead, in knowledge diffusion, individuals play two roles simultaneously: imparter and learner. If an individual has more knowledge than his or her friend, he or she would share the part of knowledge that only he or she knows with his or her friends. In a sense, knowledge diffusion seems to be closer to a heat conduction than contagion. If individuals know less than their friends, they would learn something from their friends through conversation. The difference is that an imparter does not lose its knowledge because of sharing, while an object loses its energy because of heat conduction. Thus, knowledge diffusion is a self-duplicating process. For instance, the developers of a new programming language want to popularize the technology to more users. The developers need to share their experience with the users. If the technology is helpful, these users would share their experience with each other or introduce it to their friends. Eventually, this programming language is diffusing through user networks.

On the other hand, humans cannot keep knowledge in their brains forever. Normally, knowledge in memory decays with time. Thus, a forgetting mechanism should be seriously considered when we try to further understand knowledge diffusion processes [36–38]. In general, the forgetting mechanism can be represented by a forgetting function, which is a plot of the amount remembered, $R(t)$, as a function of time since learning, t. In 1885, Ebbinghaus [39] proposed the function $R(t) = 100 \cdot \frac{a}{log^b(t)+a}$, whereas Wickelgren [40] argued that forgetting functions are better described by a power law $R(t) = at^{-b}$. Wixted and Ebbesen [41, 42] suggested that both the logarithmic and power functions are suitable for characterizing some forgetting functions, while exponential functions do not perform well when fitting the investigated empirical data. Although Anderson and Schooler [43] provided more evidences to support that the power law is a better choice, White [44] argued that the exponential-power function should be a reasonable option as well, which is confirmed by Rubin and Wenzel [45] with a short-term memory data set.

In this paper, we propose a knowledge diffusion model, where two mechanisms in cognitive psychology, forgetting and learning, are considered. We adopt a modified exponential function as the forgetting function. Knowledge is partially forgotten by individuals on one hand and consolidated by learning from their neighbors in networks on the other hand. We investigate the influence of topological structures on diffusion efficiency, characterized by the maximum of the total knowledge

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of a population. We will show that the degree heterogeneity [46] of the networks can effectively promote knowledge diffusion in complex networks.

2. RELATED WORKS

Complex networks are composed by a massive population of nodes. The connections among them are neither regular nor random [8–11], because the mechanisms forming the networks are neither simple nor unique. The complex structures bring researchers a knotty and urgent problem, how to describe complex networks. To this end, researchers define a number of statistical topological properties, in which degree distribution, clustering coefficient, and average path length attracted most attention [14, 47].

2.1. *Degree distribution*

The degree (or connectivity) k_i of a node i is the number of links connected with the node. It is defined as $k_i = \sum_{j \in n} a_{ij}$, where $n \equiv n_1, n_2, \ldots, n_N$ are the nodes of the network. a_{ij} is the entry of the adjacency matrix of the network. The degree distribution $P(k)$ is one of the most important and basic statistical property of a network [47]. By definition, the degree distribution $P(k)$ is the probability that a randomly selected node has exactly k links.

2.2. *Clustering coefficient*

The clustering coefficient provides a measure of the level of cohesiveness around any given node. A quantity C_i (the local clustering coefficient of node i) is first introduced to measure how likely two neighbors of node i are connected. By definition, the clustering coefficient C_i [47] of node i is the ratio between the number of edges E_i, which actually exist among the k_i neighbors of node i and its maximum possible value, $\frac{k_i(k_i-1)}{2}$, that is, $C_i = 2\frac{E_i}{k_i(k_i-1)}$. The average clustering coefficient C of the whole network is the arithmetic average of C_i, that is $C = \frac{C_i}{N}$, where N is the size of the network.

2.3. *Average path length*

The shortest paths play an important role in the transport and communication within a network. Suppose one needs to make a call to a friend through the telephone net: the geodesic provides an optimal path, because one would achieve a fast transfer and save system resources. For such a reason, the shortest paths have also played an important role in the characterization of the internal structure of a graph [14].

The path length of a pair of nodes is defined as the shortest distance (the length of the shortest path) between them, which characterizes the communication delay in the network [48]. A measure of the typical separation between two nodes in the graph is given by the average path length, also known as the characteristic path length, defined as the mean of geodesic lengths over all pairs of nodes, $D = \frac{\sum_{i,j \in n, i \neq j} d_{ij}}{N(N-1)}$.

Degree distribution, clustering coefficient, and average path length are the top three highly concerned topological properties. Apart from them, degree heterogeneity [46] and degree correlations [49] of a network are also extensively adopted to quantitatively characterize complex networks in another sense.

2.4. *Degree heterogeneity*

Degree heterogeneity is a statistical property, which quantitatively characterizes the fluctuation of the degree sequence of a network. This property was frequently mentioned, while its explicit definition was not manifested until a recent work [46]. To be consistent with previous studies, we denote the degree heterogeneity by H. H is defined as $\frac{\langle k^2 \rangle}{\langle k \rangle^2}$, where $\langle k \rangle$ denotes the average degree.

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2.5. *Degree correlations*

Real-world scale-free networks normally possess various degree correlations. For instance, in social networks, hubs are likely to be connected, which is regarded as an assortative mixing. In technological and biological networks, hubs are bridged by less-connected nodes, which is regarded as a disassortative mixing [49]. To measure and compare degree correlations, Pearson coefficient is introduced into the studies on network structures. A positive (negative) Pearson coefficient indicates that the network is assortative (disassortative) mixing. When Pearson coefficient equals 0, the network is degree-uncorrelated mixing.

Another interesting measure related to degree correlations is the average degree of the nearest neighbors for nodes with degree k, denoted by $k_{nn}(k)$ [50, 51], namely, $k_{nn}=\sum_{k'} k' p(k'|k)$. When $k_{nn}(k)$ increases with k, it means that nodes have a tendency to connect to nodes with a similar degree. In this case, the network is defined as assortative [49, 52]. In contrast, if $k_{nn}(k)$ decreases with k, which implies that nodes of a large degree are likely to have neighbors with a small degree, then the network is ascribed to the disassortative [53]. If correlations are absent, $k_{nn}(k) = const$.

2.6. *Learning and forgetting mechanism*

In our model, individuals play two roles simultaneously: imparter and learner. At a certain time step, a conversation takes place between two friends i and j in a network. Let $\Psi_i(t)$ be individual i's knowledge after t time steps. If $\Psi_i(t)$ is larger than $\Psi_j(t)$, i is the imparter and j is the learner at time t and vice versa. In the conversation, individual i would share the part that only he or she knows with his/her friend. In this case, the learning ability of individual j is characterized by a parameter Δ_j, which is defined as the assimilated fraction of the incoming knowledge. Simultaneously, each individual forgets a fraction of their knowledge. Let ϵ_i be the forgotten fraction of individual i to model the exponential decay of a short-term memory.

The evolution process of our knowledge system is illustrated in the following. For an individual i at time t, the value of his or her knowledge will decay $\epsilon_i \Psi_i(t)$ after a time step. In the meantime, two friends, say j and k, at the two ends of a randomly chosen link have a conversion. Take history lessons in the secondary school as an example, individual k who had this course would forget the exact year of a historical event a couple of years later, while he or she remembers the rest of the event. Individual j may even forget the location of the event. A conversation between them may remind individual j from the second group of the location. In this case, $\Psi_j(t) < \Psi_k(t)$, j will assimilate $\Delta_j |\Psi_k(t) - \Psi_j(t)|$ after a time step.

3. MULTICHANNEL KNOWLEDGE DIFFUSION IN COMPLEX NETWORKS

To focus on the dynamical process, we ignore the differences on the learning ability and retention among individuals and set $\Delta_i = \Delta$ and $\epsilon_i = \epsilon$ for all i. To compare the influence of different topological structures on knowledge diffusion, we define the total knowledge of the population $\Phi(t) = \sum_i \Psi_i(t)$, where $\Psi_i(t)$ denotes individual i's knowledge after t time steps. In a network, we initially set $\Psi_i(0)$ to a random value ranging from 0 to 1.

3.1. *Evolution of total knowledge*

Based on the assumptions mentioned earlier, the evolution of the total knowledge $\Phi(t)$ satisfies the following difference equation:

$$\Phi(t) = (1-\epsilon)\Phi(t-1) + \Delta \times Z(t), \tag{1}$$

where $Z(t) = |\Psi_i(t) - \Psi_j(t)|$. Here, i and j denote the two ends of a randomly picked link at the step t. This equation indicates that knowledge diffusion is directed, which is a downhill diffusion.

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Through a conversation between two friends in a social network, an individual i with a smaller $\Psi_i(t)$ may learn something from individual j through the conversation. Solving Equation (1), one derives

$$\Phi(t) = (1-\epsilon)^t \Phi(0) + \Delta \sum_{s=0}^{t} (1-\epsilon)^{t-s} \times Z(s), \tag{2}$$

Because $Z(s)$ has a upper bound $(1-\epsilon)^s$, we have the following inequality

$$\Phi(t) \leq (1-\epsilon)^t \left(\Phi(0) + \Delta(t+1)\right). \tag{3}$$

The term of the right-hand side is the upper bound of the total knowledge $\Phi(t)$. One can see that $\Phi(t)$ would decay with t in the end, because the rate of decay in Equation (3) is exponential.

3.2. Average knowledge of the individuals with a certain degree

For the networks with degree heterogeneity, for instance, the Barabási–Albert scale-free network (BASN) [47] and star network, we define $\omega_k(t)$ as the average knowledge of individuals with degree k after t steps. Let $\alpha_k(t)$ be the learning probability for an individual with degree k after t steps. The evolution of $\omega_k(t)$ in a network with clear degree heterogeneity satisfies

$$\omega_k(t) = (1-\epsilon)\omega_k(t-1) + \frac{\alpha_k(t-1)\Delta Z(t) P(k|k')}{P(k)N}, \tag{4}$$

where $P(k|k')$ denotes the conditional probability of that an individual with degree k' connects with an individual with degree k. Here, k and k' denote the degrees of both ends of the picked link at time t, respectively. N denotes the size of the network. For the degree-uncorrelated scale-free networks [49], for instance, the BASN [47],

$$P(k|k') = \frac{P(k)k}{\langle k \rangle}, \tag{5}$$

where $P(k)$ denotes the probability of an individual having degree k, which is also known as the degree distribution of a network. $\langle k \rangle$ denotes the average degree. Because $P(k|k')$ is irrelevant to k', Equation (4) can be rewritten as

$$\omega_k(t) = (1-\epsilon)\omega_k(t-1) + \alpha_k(t-1)\Delta Z(t) \frac{k}{\langle k \rangle N}. \tag{6}$$

Respecting our initial settings, $\omega_k(0) = F(P(k)N)$, where $F(.)$ is a function of $P(k)N$. $F(P(k)N)$ is close to $\frac{1}{2}$ for $P(k)N \gg 1$, while it is a random value ranging from 0 to 1 when $P(k)N$ is close to 1. Based on these initial conditions, the individuals with more friends in the network generally have a higher probability to learn some knowledge from their friends at the beginning, because $\alpha_k(0) \simeq \frac{1}{2}$ in Equation (6) for $P(k)N \gg 1$.

3.3. Learning probability for the individuals with a certain degree

This advantage of the well-connected individuals result in that their learning probability decays with t, while the learning probability of the less-connected individuals grows with t accordingly. Thus, one can roughly predict $\alpha_k(t)$ when $t \to \infty$ by summarizing the probability of connecting with individuals with larger degrees. If the chosen link happens to exist between the two individuals with the same degree k, $\alpha_k(t) \simeq 1$ because the system can hardly reach a complete synchronized status. Based on this basic understanding, one can approximately obtain

$$\alpha_k(t) = \sum_{k'=k}^{k_{max}} P(k'|k), \tag{7}$$

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In the degree homogeneous network, such as regular graphs, the Watts–Strogatz small-world network (WSSN) [14], and random graphs, one can consider the scenario as that an individual with $\langle k \rangle$ plays with $\langle k \rangle$ individuals with $\langle k \rangle$. $\alpha_k(t)$ for all k is generally stabilized around 0.5. The small difference of $\alpha_k(t)$ among individuals with different degrees also originates from the weak degree heterogeneity.

For the star network, because there is only one hub with a degree $N-1$. Although $\omega_k(0)$ is a random value ranging from 0 to 1, it is likely to learn more knowledge from their friends at the beginning of evolution, because its probability to be chosen is 1. As a result, the individuals surrounding the hub can benefit from the hub later because they have an erudite neighbor. In this respect, the existence of this extreme degree heterogeneity highly promotes knowledge diffusion in this class of networks. Given Equation (7), one can derive $\alpha_{N-1}(\infty) = 0$ and $\alpha_1(\infty) = 1$.

4. EXPERIMENTAL RESULTS

4.1. Experimental settings

To clarify the influence of topological structures on knowledge diffusion, we then run extensive numerical simulations on the BASN, regular network, WSSN, random network, and star network. We initially generate a network of 1024 individuals with random seeds. In the network, we assign a random value ranging from 0 to 1 to each individual as his or her knowledge value. Next, we randomly choose two friends i and j in the network to have a conversation. In this conversation, an individual i with more knowledge, namely $\Phi_i(t) > \Phi_j(t)$, would distribute his or her knowledge to j. In our simulations, all the individuals are equally talented, that is, $\Delta_i = \Delta$ and $\epsilon_i = \epsilon$, where $i = 1, 2, \ldots, N$. Because our purpose is to test the influence of the topological structures, we uniformly set $\epsilon = 0.0001$ and $\Delta = 0.5$ for all the individuals in our simulations.

4.2. Average knowledge of the individuals with a certain degree

Figure 1 shows the evolution of $\omega_k(t)$ on the BASN, regular network, WSSN, random network, and star network, where 1024 individuals in (a), (c), (e), (g), and (i) (4096 individuals in (b), (d), (f), (h), and (j)) are coupled. One can observe that $\omega_k(t)$ grows uniformly at the beginning. Comparing the diffusion processes shown in Figure 1(a) and (b) with the other panels, one can observe that the individuals with more friends in the network can rapidly learn more knowledge before $\omega_k(t)$ decays with time. The relation between the maximum of $\omega_k(t)$ and k is shown in the inset. In (a), the curves from bottom to top correspond to the evolutions of $\omega_k(t)$, where k increases from 3 to 123. In (b), the curves from bottom to top correspond to the evolutions of $\omega_k(t)$, where k increases from 3 to 229. This observation originates from $\alpha_k(t)k$ in Equation (6). Let $\alpha_k(t)$ be the probability that the individuals with degree k can learn knowledge from their friends after t steps. Initially, $\alpha_k(t)k$ of hubs is much larger than that of the individuals with small degree. As mentioned previously, all the individuals' knowledge is randomly assigned. Thus, $\alpha_{k_i}(0) \simeq 0.5$ for all i. For an individual i with a large degree, for instance $k_i = Max_j(k_j)$, $\alpha_{k_i}(t)$ quickly decays with time as shown in Figure 2. In (b), (c), (d), (e),(f), (g), (h), (i), and (j), the relations between $\omega_k(t)$ and k are similar to the observation in (a), that is, the curve at the top is likely to correspond to $\omega_{k_{max}}(t)$. Because the degree heterogeneity of the WSSNs and random networks is much smaller than the BASNs and star networks, the distance of $\omega_k(t)$ between the highly connected and less-connected in (e), (f), (g), and (h) is not so evident as that in (a), (b), (i), and (j). Apart from degree heterogeneity, one can see that the behavior is neither relevant to the size of a network nor to the type of the network.

4.3. Learning probability of individuals with a certain degree

Figure 2 shows the evolution process of $\alpha_k(t)$, which represents how likely an individual with degree k would learn something through a random conversation with one of his or her friends. Clearly, if $\alpha_k(t) > 0.5$, an individual with degree k is likely to learn more knowledge at time t. In contrast, $\alpha_k(t) < 0.5$ means that the individuals have more knowledge than most of their friends, who would probably diffuse their knowledge to their neighbors in the next steps. In (a) and (b), the learning

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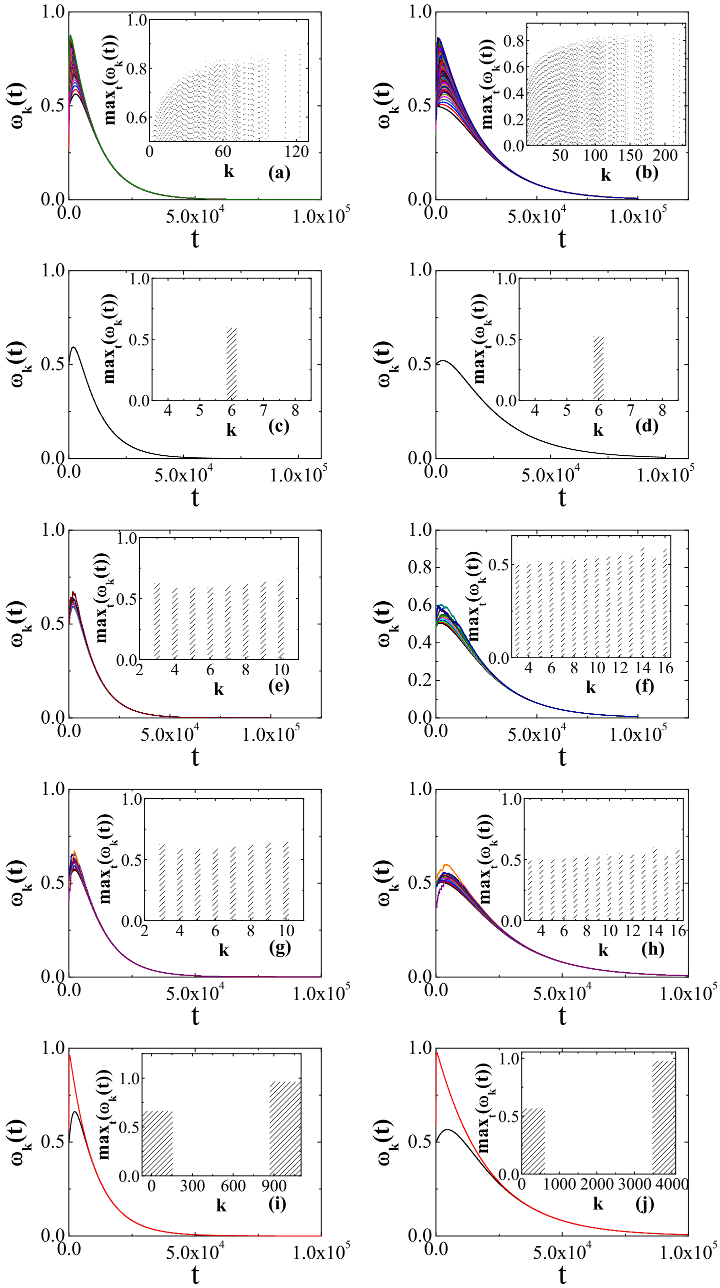

Figure 1. The evolution process of $\omega_k(t)$ on five topological structures. We test the case for $\alpha = 0.0001$ on the Barabási–Albert scale-free network (BASN) shown in (a) and (b), regular network shown in (c) and (d), Watts–Strogatz small-world network (WSSN) shown in (e) and (f), random network shown in (g) and (h), and star network shown in (i) and (j). The networks in (a), (c), (e), (g), and (i) are composed of 1024 individuals. We also test the case for $\alpha = 0.00005$ on (b) the BASN, (d) regular network, (f) WSSN, (h) random network, and (j) star network composed of 4096 individuals. In this figure, Δ is set to 0.5. We set $\alpha = 0.00005$ in the networks with 4096 individuals, because the total knowledge $\Phi(t)$ directly decays with time for $\alpha = 0.0001$. The BASNs are generated by $m_0 = m = 3$ [47], where m_0 denotes the size of the initial fully connected network and m denotes the number of links among a new individual and the existing individuals in the network. The WSSNs and random networks are generated by randomly rewiring 10% and 100% of the links in the regular network, which are formed by 1024 (4096) identical individuals of degree 6. The star network is composed by one individual of degree 1023 (4095) and 1023 (4095) individuals of degree 1. The simulation results were obtained by averaging 10 diffusions on 10 different realizations of the same type of network specified by the appropriate parameters. Thus each plot in this figure corresponds to 100 simulations.

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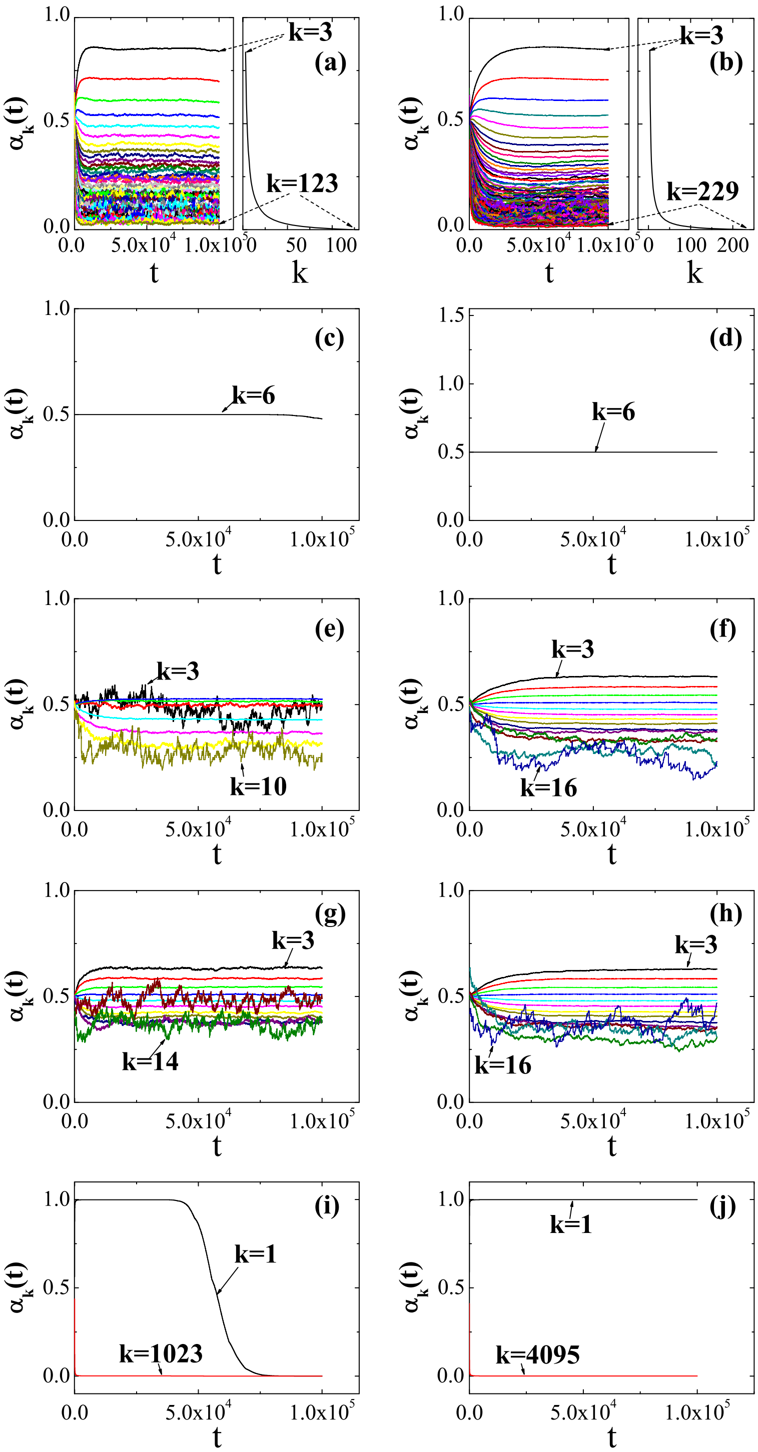

Figure 2. The evolution process of $\alpha_k(t)$ on five topological structures. All the simulation settings are consistent with Figure 1. In (a) and (b), the curves from top to bottom correspond to the evolutions of $\alpha_k(t)$, where k roughly increases from 3 to 123 in (a) (from 3 to 229 in (b)).

probability of highly connected individuals decays rapidly, while that of the less-connected grows in the meantime. The values of $\alpha_k(t)$ for the highly connected individuals reach their minimum before that of the less-connected reach their maximum. After reaching the minimum, $\omega_k(t)$ for all k roughly stabilize at the minimum with small fluctuation. The observation confirms our explanation

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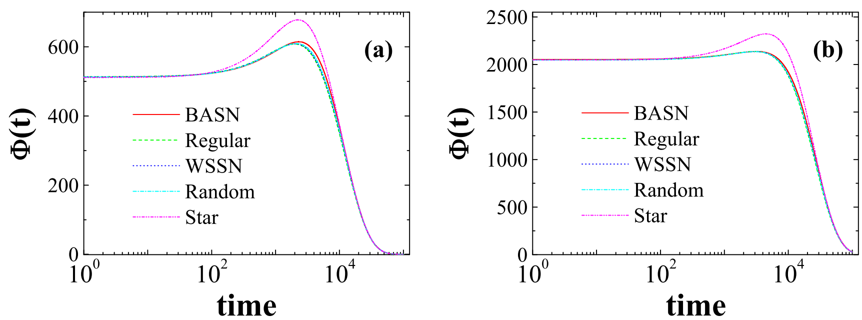

Figure 3. Evolution of the total knowledge $\Phi(t)$ on the five topological structures. The top curve shows the simulation result obtained in the star networks. The second one shows the corresponding results in the Barabási–Albert scale-free networks. The results in the regular networks, Watts–Strogatz small-world networks, and random networks are basically overlapped at the bottom.

on the simulation results shown in Figure 1 in another sense. Given Equation (5), one can derive an analytical solution of Equation (7), which is shown in the inset. In (e) and (g) ((f) and (h)), the relations between $\alpha_k(t)$ and k are consistent with the observation in (a) ((b)). One can see that the observation in (j) confirms our analytical prediction about $\alpha_{N-1}(\infty) = 0$ and $\alpha_1(\infty) = 1$ in star networks in Section 3. In (i), because the distance of knowledge among the hub and its neighbors become very small after 5×10^4 steps, the system gradually gets into a quasi-synchronized state, where the distance among Ψ_i for all i can hardly be differentiated in double precision. In this case, $\alpha_k(t) \to 0$ for all k.

4.4. *Total knowledge*

Figure 3 shows the evolution of $\Phi(t)$ on all the topological structures mentioned earlier. One can observe that $\Phi(t)$ in the networks gradually grows to the maximum first. After reaching the maximum, $\Phi(t)$ rapidly decays with time. In terms of the maximum of $\Phi(t)$, the maximum of $\Phi(t)$ in the star network is the largest. For the rest, the BASN is slightly higher than the regular network, WSSN, and random network, which confirms that degree heterogeneity is beneficial to knowledge diffusion in another sense. Except the star network and BASNs, all the traces of $\Phi(t)$ in the other three topologies are almost identical, because the degree heterogeneities of the three topologies are much smaller than those of the star networks and BASNs. In another sense, this observation is interesting, since structures of social networks are demonstrated to have a nontrivial influence on the epidemic-like information dissertation, such as rumor spreading [24, 26]. In the previous studies, researchers found that the final informed scale in scale-free networks is smaller than that in the WSSNs [24] with the same simulation average degree. Our observation, however, suggests that knowledge as a familiarity, awareness, or understanding of some information can spread more efficiently in the scale-free networks. Here, the efficiency is measured by the maximum of $\Phi(t)$. Although the star network is the most suitable structure for knowledge diffusion, most real social networks are not star-like, because each individual has a Dunbar number [54]. Thus, the most practical organization in the mentioned structures is the scale-free networks, which happens to be consistent with the structures of most realistic social networks [8–11].

4.5. *Influence of topological properties*

For the networks investigated previously, their topological properties are clearly different from each other. For example, the degree distribution of the BASN follows a power law [47], while that of the WSSNs follows a Poisson distribution [14]. However, from the perspective of degree distribution, one can hardly conclude which type of distribution is suitable for knowledge diffusion. Likewise, the clustering coefficient and average path length [14] of the WSSNs, random networks, and regular networks are clearly different (Table I), while the peaks of $\Phi(t)$ for the three networks are very close. These observations indicate that the classical topological features are not sufficient to govern the performance of knowledge diffusion in a network. After testing a series of topological features, we found that the degree heterogeneity [46] is likely to be the most representative one.

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Table I. Clustering coefficient, average path length, and degree heterogeneity of the networks investigated in Figure 3.

		SN	BA	Reg	WS	Ran
(a)	Clustering coefficient	0	0	0.6	0.45	0
	Average path length	2	3.47	85.75	6.19	4.12
	Degree heterogeneity	256.25	2.326	1	1.015	1.051
(b)	Clustering coefficient	0	0	0.6	0.44	0
	Average path length	2	3.98	341.75	7.66	4.93
	Degree heterogeneity	1024.25	2.747	1	1.016	1.012

SN, star networks; BA, BASNs; Reg, regular networks; WS, WSSNs; Ran, random networks.
(a) shows the results of the networks with 1024 individuals and (b) shows the results of the networks with 4,096 individuals.

As shown in Table I, one can observe that the peak of $\Phi(t)$ is exactly proportional to H. Even for the case that the distances among peaks in regular networks, the WSSNs, and random networks are very small, one can likewise differentiate them by H. For instance, Figure 3(a) shows that the peaks of $\Phi(t)$ in regular networks and the WSSNs with 1024 individuals are 607.97 and 608.26, respectively. In Table I, one can observe that the degree heterogeneity value $H = 1.015$ for the WS networks, which is also slightly greater than $H = 1$ for the regular networks.

Table I shows that the degree heterogeneity of the BASNs is clearly higher than other non-star networks, which indicates that scale-free networks are suitable and practical for knowledge diffusion in humans. The BASNs, however, are not necessarily the most suitable scale-free networks for knowledge diffusion. To clarify which type of scale-free networks is more suitable for knowledge diffusion, we also test the degree heterogeneity of a series of scale-free networks. In the BASN, the probability that a new individual j connects with an existing individual equals $\frac{k_j}{\sum_i k_i}$. We adjust the probability to $\frac{k_j^\delta}{\sum_i k_i^\delta}$, where δ is a constant. One can see that H grows with δ monotonously in Figure 3(a) and (b). Our observation indicates that the scale-free network with a high-degree heterogeneity can further promote the performance of knowledge diffusion. Figure 4(c) and 4(d) show the degree distributions of the scale-free networks. This result confirms our previous conclusion derived from comparison among five different types of networks, which is that the degree heterogeneity or polarization is beneficial to knowledge diffusion.

For integrity, we also investigate the influence of degree correlations on knowledge diffusion in scale-free networks. Because the Pearson coefficients of most real-world networks fall in the region of $[-0.3, 0.3]$ [49], we adopt the XS algorithm [55, 56] to generate scale-free networks with degree correlations. We test the scale-free networks with assortative mixing, degree-uncorrelated mixing, and disassortative mixing, respectively. Not unexpectedly, Figure 5 shows that degree correlations are completely irrelevant to knowledge diffusion.

Knowledge diffusion is a special form of information spreading. We investigate knowledge diffusion in complex networks, because most existing models [22–35] on information spreading, as far as we concerned, can hardly characterize the unique interaction mechanism in knowledge diffusion. The existing models normally take the propagating information as an epidemic, while knowledge is clearly different from an epidemic, because individuals in a knowledge diffusion network are willing to learn and spread the knowledge.

On the other hand, in terms of the studies on memory, disregarding humans' learning capacity and social property may bring the researchers a misleading conclusion. In this paper, we summarize these factors and propose such a simple model, committed to shedding some light on the difference between the star-like one-to-many school education and complex scenarios such as online self-education and international conferences. We believe that a reasonable organization of individuals may highly promote the effectiveness of knowledge diffusion when their abilities are fixed.

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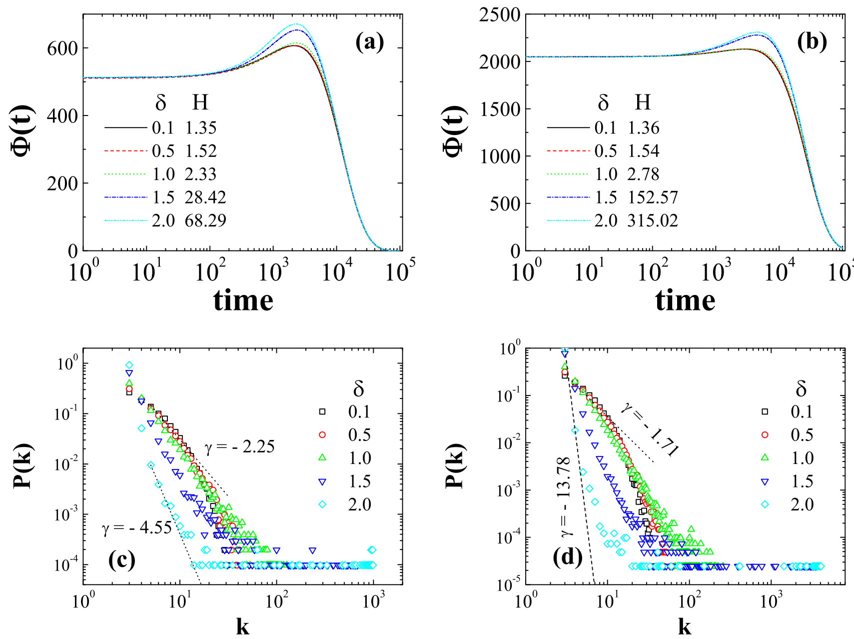

Figure 4. Evolution of the total knowledge $\Phi(t)$ on the scale-free networks with different degree heterogeneities. (a) and (b) show the simulation results of the scale-free networks with 1024 and 4096 individuals, respectively. We test the cases of $\delta = 0.1, 0.5, 1.0, 1.5$, and 2.0. (c) and (d) show the degree distributions of the networks. Note that (c) and (d) are two log–log graphs. We fit the curves by $P(k) = Ak^{\gamma}$, where A and γ are two constants. One can see that the values of γ for the tested scale-free networks are confined to $[-4.55, -2.25]$ in (a) ($[-1.71, -13.78]$ in (b)).

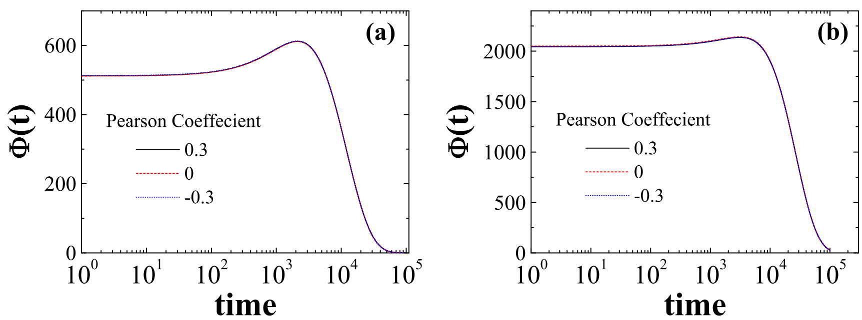

Figure 5. Influence of degree correlations on knowledge diffusion. We show the cases that Pearson coefficient equals 0.3, 0, and -0.3, respectively. (a) and (b) show the case of $\alpha = 0.0001$ with 1024 individuals and $\alpha = 0.0005$ with 4096 individuals, respectively. In these networks, we uniformly set $\delta = 1.0$ and $\Delta = 0.5$.

5. CONCLUSION

In a nutshell, we have proposed a model to simulate knowledge diffusion process in complex networks. In this model, two mechanisms in cognitive psychology, forgetting and learning, are considered. The diffusion process is directed, which follows a from-high-to-low rule. Briefly, if an individual has more knowledge than his or her friend in a conversation, s/he would share the part of knowledge that only he or she knows with his or her friend. If he or she knows less than his or her friend, instead, he or she would learn something from his or her friend accordingly. In this scenario, our results show that the individuals with a large degree in a network with clear degree heterogeneity are likely to learn more knowledge than their counterparts with smaller degrees at the early stage of knowledge diffusion. The prediction is confirmed by extensive numerical simulations. On the other hand, we observe that the star network is the most efficient platform for knowledge

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diffusion. As the second efficient platform, the spreading efficiency of the BASNs is higher than that of the WSSNs. In scale-free networks, we observe that the degree heterogeneity is beneficial to knowledge diffusion. Interestingly, this observation is in contrast to the conclusion of an epidemic-like information dissemination. In the epidemic-like information dissemination, more individuals in the WSSNs are informed when the average degrees for both types of networks are equal.

Knowledge diffusion possesses two special properties, gradient self-duplication and positive response from individuals, which makes it different from epidemic-like information-spreading processes. As far as we know, the studies targeted on knowledge diffusion in complex networks are limited. Thus, there is a need to further investigate such a special form of information dissemination. In this paper, we propose a simple theoretical model for providing a better understanding of knowledge diffusion on complex networks. Acknowledgedly, there are some other mechanisms, which may be as crucial as the mentioned. Because our focal topic in this paper is the influence of topological structures on knowledge diffusion, they are simplified in our model. We believe, however, that both the advantages and disadvantages of this model may be helpful for the studies of knowledge diffusion in the future.

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