

# ROBUST EXPONENTIAL STABILITY OF UNCERTAIN SWITCHED STOCHASTIC SYSTEMS WITH TIME-DELAY

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**Abstract.** The abstract should be informative, precise and not exceed 200 words. Please do not include equations, tables and references in the abstract.

**Keywords.** At least 5 key words should be provided.

## 1 Introduction

Authors are required to state clearly the contributions of the paper in the Introduction. There should be some survey of relevant literature.

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Stochastic dynamic modeling plays an essential role in numerous physics and engineering problems. It can be applied wherever random properties of a dynamical system have to be considered. Most of the emphasis is placed on the stability analysis of the stochastic dynamical systems (see [1]). Moreover, in many applications, the physical or chemical processes are governed by more than one dynamics: the dynamics change among a family of choices with respect to time  $t$  or state  $x$ . Such

processes are often described by switched systems and have been studied extensively in recent years (see [5, 6]). Time-delay and uncertainties are two main causes for instability of dynamical systems (see [2]). Numerous studies have been carried out on stability analysis and stabilization of time-delay systems and uncertain systems (see [3, 4]), some of which have been done in the scope of stochastic systems or switched systems. To the best knowledge of the authors, few work has been done for switched stochastic systems with both uncertainties and time-delay. Some results are given by the figure 1.

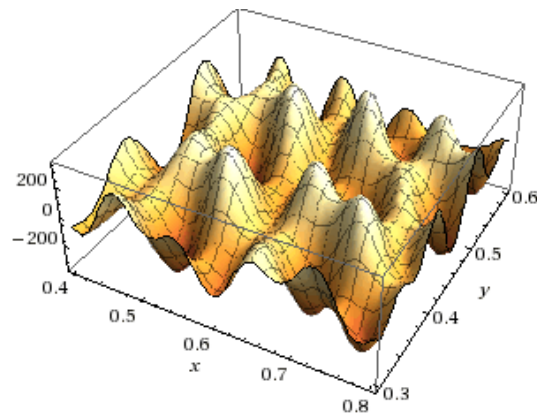


Figure 1: A beautiful surface

## 2 Problem statement and preliminaries

### 2.1 Formulation

Consider the following stochastic uncertain switched system

$$\begin{aligned} dx(t) = & [(A_i + \Delta A_i)x(t) \\ & + (\tilde{A}_i + \Delta \tilde{A}_i)x(t-h)]dt \\ & + [(B_i + \Delta B_i)x(t) \\ & + (\tilde{B}_i + \Delta \tilde{B}_i)x(t-h)]dw(t), \end{aligned} \quad (1)$$

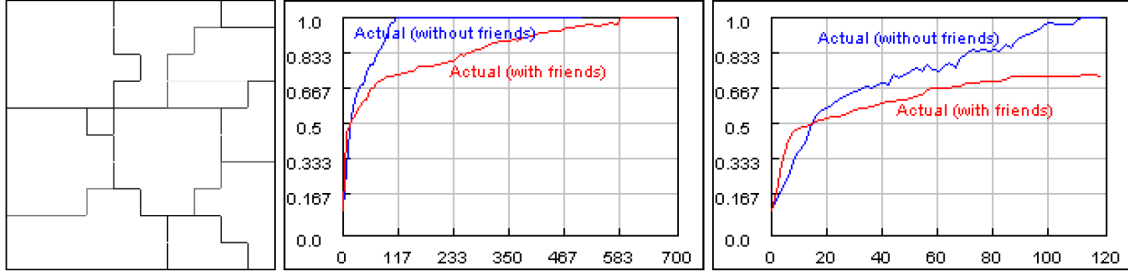
$x(t) = \phi(t), t \in [-h, 0]$ ,

where  $x \in \mathbb{R}^n$  is the state and  $h$  is the constant time-delay.

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**Figure 2:** In this first experiment, typical outcome for a modified homogeneous Axelrod model is presented. (Left) An intermediate configuration in the case of partner selection. (Middle) Actual affinities with and without friends (partner selection). (Right) Zoom to the first part of the previous graph (first 120 cycles).

In the following, we describe some experiments.

**Experiment 1.** Modified Axelrod model (homogeneous) results with an initially diverse population (fig. 2). Partner selection does not change the final outcome which is full monoculture in the population, but it substantially retards the cultural contagion process. On the other hand, partner selection induces extremely fast local polarization as is depicted in fig. 2(left) where an intermediate configuration is shown, that is not possible without partner selection in this model. Fig. 2(right) shows the large speed of local convergence in the beginning of the experiment that slows down in what follows compared to the convergence speed without partner selection.

**Experiment 2.** Modified Axelrod model (homogeneous) with two initial populations where a cultural clash is expected. Again, partner selection does not change the final outcome which is full monoculture in the population, but it substantially retards the cultural contagion process. This is due to the border agents between the two populations that create individualistic partnerships and thus hinder the fast spreading of cultural information.

## 2.2 Stability

Two types of stability are considered in this paper, one is almost sure exponential stability and the other is exponential stability in  $p$ th moment (see [5] for definitions). The main problem now can be formulated as follows:

**Problem 2.1.** *For all admissible uncertainties, under what conditions will the uncertain switched system (1) be almost surely exponentially stable? Under what conditions will it be exponentially stable in mean square?*

## 3 Main results

In this section, the stability analysis of system (1) is studied.

**Table 1:** Stability bounds of time-delay and average dwell time

$T_0(\times 10^3)$	0	0.0856	0.2303	0.7960
$h_0$	0	0.5	1.0	1.5
$T_0(\times 10^3)$	1.7645	3.0717	5.1803	9.2734
$h_0$	2.0	2.5	3.0	3.5
$T_0(\times 10^3)$	23.4052	238.2837		
$h_0$	4.0	4.4		

The following matrix inequalities are satisfied:

$$\psi_i = \begin{bmatrix} \psi_{11} & \psi_{12} & \psi_{13} \\ * & \psi_{22} & \psi_{23} \\ * & * & \psi_{33} \end{bmatrix} < 0, \quad (2)$$

$$Q_i \leq \rho_i I, \quad (3)$$

## 4 Application

For different average dwell time lower bounds  $T_0$ , the delay upper bounds  $h_0$  guaranteeing the exponential stability of the system are listed in Table 1.

## Acknowledgment

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