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## MODELING THE DYNAMICS OF COMPLEX INTERACTION SYSTEMS: FROM MORPHOGENESIS TO CONTROL

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The aim of this paper is to contribute to the modeling and analysis of complex systems, taking into account the nature of complexity at different stages of the system life-cycle: from its genesis to its evolution. Therefore, some structural aspects of the complexity dynamics are highlighted, leading (i) to implement the morphogenesis of emergent complex network structures, and (ii) to control some synchronization phenomena within complex networks. Specific applications are proposed to illustrate these two aspects, in urban dynamics and in neural networks.

*Keywords*: Complex systems; dynamical systems; complex networks; self-organization; synchronization; network topology.

## 1. Introduction

#### 1.1. Context

The complexity of some dynamical systems has been studied since several decades. As an example, multiscale properties of hydrodynamic instabilities have been observed since mid twentieth century, in the early studies of turbulence. The description of multiscale turbulence has been defined by Richardson [1922] and then formulated with Kolmogorov cascade [Kolmogorov, 1941] as early work on statistical physics theory. The observations and analysis of complex properties have been developed in many scientific domains according to their context, in relation to the feedback process [Brian Arthur, 1990] or with market crash [Sornette, 2003], in geopolitics [Cederman, 1997], in city development [Batty & Longley, 1994; Batty, 2005] where emergent scaling laws can be observed [Pumain *et al.*, 2007] or with seggregative urban pattern formation [Schelling, 1971], but also in biology with collective formation of bacteria [Ben-Jacob & Levine, 2001], of birds [Reynolds, 1987] and with self-organized process [Camazine et al., 2001]. New approaches to the Science of Complexity — to which such a large number of scientists claim to belong nowadays is mainly due to two important aspects. The first one is to identify specific properties of complexity, crossing a great variety of disciplinary fields: from biology to sociology or linguistics [Erdi, 2008]. The second aspect is to propose some mechanisms of complex system reconstruction, preserving the complexity itself. A reconstruction, as any conceptual approach, does not avoid reductionist touches within the engineering steps, commonly opposed to the holistic approach of complexity. The great challenge is therefore to preserve the complexity of the system through its recreation. That means the elementary features of the system complexity need to be identified and the complexus of links among these features need to be created, allowing the whole to emerge as the essential complex property of the system. Generally, these elementary features of complexity find their roots in interdisciplinary concepts, making them transversal to many scientific domains.

## 1.2. Complexity of dynamical systems through structural properties

One of the main characteristics of complexity is the emergence of properties due to dynamical processes. Our objective is to contribute to the formalization of these emergent properties studying dynamical structures. The complexity dynamics is not only a one-way expression of the structure properties, but the structure itself controls the dynamics of the whole. The structures of complexity proposed here, are interaction systems at the core of selforganization mechanisms. During morphogenesis or more generally along morphodynamics, the structure topology is emergent or evolving [Bourgine & Lesne, 2006].

Dynamical networks are efficient tools to express some local or global properties of this evolving topology. They capture structural aspects of complex systems representing entities as nodes and interactions between them as links. Empirical studies of different networks, such as Internet and World Wide Web [Pastor-Satorras & Vespignani, 2004], metabolic and protein networks [Jeong et al., 2000, 2001], collaboration networks [de Castro & Grossman, 1999; Watts & Strogatz, 1998] and many others, have shown that despite their diversity, these networks share common topological features. Such features include degree distribution, clustering, shortest path lengths and betweenness, community structure, etc. Different classes of complex networks have been proposed and largely studied, including small-world networks [Watts & Strogatz, 1998; Watts, 1999; Buchanan, 2002] and scalefree networks [Barabási & Albert, 1999; Barabási, 2003; Caldarelli, 2007]. Different growth models have been proposed [Albert & Barabási, 2002; Dorogovtesev & Mendes, 2003] which mimic the morphogenesis of real-world networks. There are many experimental and analytical evidences that the network topology crucially influences essential network properties, such as resilience and tolerance to attacks [Albert et al., 2000; Cohen et al., 2001], spreading processes [Newman, 2002; Pastor-Satorras & Vespignani, 2004], but also the collective dynamics phenomena, such as self-organization, traveling waves and synchronization [Wu, 2007; Barrat *et al.*, 2008]. For extensive lists of references on complex networks, see for example [Newman, 2003: Boccaletti et al., 2006].

In this paper, we focus on the implementation of the dynamic aspects within the reconstruction of complex systems, which essential property is to be open, i.e. permanently crossed by energy fluxes. Two major aspects of the dynamics characterize the complex properties of the studied system, whatever its scientific discipline. The first one concerns the constitution of the system itself: how such permanently open systems construct themselves? This first aspect corresponds to what we call morphogenesis. The second one concerns the dynamics of a system after its constitution: how does this open system evolve? What are its self-control mechanisms? What is the impact of this self-control on its complexity? Note that these two aspects, appearing during and after the constitution of the system, are closely related. Indeed, an open complex system is never completely defined or created at a given moment but is permanently rebuilding itself, by both its internal interactions and its interactions with its environment.

A complex system is characterized by all these interactions, which constitute its structure as the output of its morphogenesis. Our work focuses on the structural aspects of complex systems and more precisely on the fact that the dynamics on structures and complexity emergence are linked from their genesis to their control and their evolution. In the two following sections, we develop (i) structural morphogenesis of complex self-organized systems and (ii) dynamical control and synchronization of complex networks. Each of these two sections starts with general concepts and results and ends with specific applications as a practical illustration. The first application, based on human sciences, concerns urban dynamics while the second one, based on biology, concerns neuronal networks.

## 2. Structural Morphogenesis of Complex Self-Organized Systems — Concept and Application

In this section, our first objective is to study the genesis of complex systems through selforganization processes. Our modeling approach is then to combine elementary behaviors in order to simulate a system whose overall behavior cannot be designed by the modeler but has to emerge from local interactions and dynamical process [Bertelle *et al.*, 2009].

## 2.1. Emergent structure from local behavioral interaction on spatial systems

The aim of this study is to model and analyze social organizations. Indeed, from interactions between

individuals emerge some organizations. We focus both on interactions between individuals and interactions between individuals and organizations. Our objective is to highlight the emergent complexity based on the two main factors: (i) the spatial dimension as a major input for the component interaction, (ii) the self-organization as a major process for the system dynamics and adaptation.

In order to model the dynamics of complex interactions between individuals and organizations, we introduce two levels of description for each of them, as described in Fig. 1. The objective is to implement an adaptive mechanism for our model that is structured three-fold:

- The emergent process. Inputs are (i) the system of spatial individuals, (ii) the system of spatial organizations. The output is the spatial distribution of individuals over the emergent organizations system. The process is based on an attraction mechanism involving complex interactions between individuals and organizations.
- *The adaptive process.* The input is the spatial distribution of individuals over the emergent organizations system. The output is the response of the organizations according to their dynamic evolution. This response leads to modification of the organization characteristics and morphology.
- *The feed-back process.* The input is the new characteristics of the organizations which are produced by the adaptive process. The output is the new spatial distribution of individuals according to the organization's system evolution.



Fig. 1. Multilevel system model: interactions between individuals inside the Entity Level, lead to emergent processes and generate organization formations; interactions from organizations over the entities lead to feedback process; adaptive processes on organizations are generated in response to these two previous processes, emergent and feedback ones. Moreover, interactions between structures inside the organization level are described by complex networks.

In the following section, we first describe the bioinspired mechanism as the basis of our model, the model formalism is then described and finally, experiments and analyses are given.

#### 2.2. Bioinspired model

# 2.2.1. Natural template in collective building

Termites mounds, ant nest, bees and wasp hives are some examples of complex structures emerging from the interactions between the social insects and their spatial environment [Perna *et al.*, 2008]. One of the mechanisms used in these structures is templates. A template is a pattern existing in the environment and used to construct another pattern [Bonabeau *et al.*, 1999]. Temperature, humidity, chemical gradient, physical functions produce different kinds of natural templates leading to specific structures.

An example of template (combined with selforganization mechanism) is the one proposed by Leptothomx albipennis ants [Camazine *et al.*, 2001], which constructs simple perimeter walls in a twodimensional nest at a given distance from the tight cluster of ants and brood and which serves as a chemical or physical template (see Fig. 2). The probability of depositing a grain is higher when both the distance from the cluster is appropriate and the local density of grains is large. On the opposite, the



Fig. 2. Wall formation generated by Leptothomx albipennis ant colonies [Camazine *et al.*, 2001].

probability of depositing a grain is lower when the cluster is either too close or too far and when the local density of grains is small. When the distance from the cluster does not lie within the appropriate range, deposition can nevertheless be observed if grains are present. Conversely, if the distance from the cluster is appropriate, deposition can take place even if the number of grains is small.

## (i) Wall building model: Implementation in repast

The algorithm is based on the ant clustering algorithm which consists of simulating how ants are able to spatially classify corpses or larvea using the following elementary decentralized behavioral process. In order to extend this basic ant clustering algorithm toward nest building algorithm, we have to define a template probability  $P_t$  which will be added to the probability of picking up and dropping.

• When an ant is moving without carrying material, if it finds some, it will take it respecting the probability number [Camazine *et al.*, 2001]:

$$P_p = \left(\frac{k_1}{k_1 + f}\right)^2 (1 - P_t) \tag{1}$$

where f is the material density that the ant perceives locally around itself,  $k_1$  is a given threshold and  $P_t$  is the template function.

• When an ant is moving and carrying some material, the probability to deposit is computed by [Camazine *et al.*, 2001]:

$$P_d = \left(\frac{f}{k_2 + f}\right)^2 P_t \tag{2}$$

where f is still the material density perceived by the ant, locally around itself and  $k_2$  is another given threshold.

In Fig. 3(a), we represent a template function adapted to the natural ant wall building and we then represent [Figs. 3(b) and 3(c)] the result of a simulation made on the agent-based java platform, Repast [2009].

## (ii) Adaptive spatial organization feedback implementation

Complex systems deal not only with emergent organization processes arising from interactions of its own entities, but also with the feedback processes





(b)

Fig. 3. Simulation of the adaptive queen behavior, according to the spatial perception of its surround material. The template function is drawn in (a) and simulation results are shown in (b) and (c). An adaptive process is implemented making the queen size grow. This size corresponds to the yellow circle diameter. Two areas are defined. The first area is near the queen and materials are expected to be removed by ants from this area. The second area is a disk where materials are expected to be deposit by ants. When material number in each of these two areas reaches some threshold, the queen grows and the two area diameters are modified: (a) Template function adapted to the natural ant wall building, (b) simulation on Repast: after few steps and (c) Simulation on Repast: after queen adaptive development.

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Fig. 3. (*Continued*)

of the organization over its components. In the proposed model, we can take into account such feedback processes and we present in Fig. 3 an adaptive process which makes the queen (describing the organization itself) modify the environment and the clustering processes. Following the template function, the queen locally defines her two zones. The first zone is near the queen and no material is expected there. The second zone corresponds to the template maximum and a great concentration of material is expected there. In the simulation, we count in a dynamical way the number of materials in these two zones and when these numbers reach some thresholds, the queen evolves by increasing its own size and hence increasing the two associated zones. After this evolution, ants have to move some material following the new template function attraction. The lower part of Fig. 3 shows the evolution of the queen which has evolved six times since the simulation beginning. In this figure, we can see the red curves counting the zones density. Each gap in these density curves corresponds to an evolution of the queen.

## 2.2.2. Multi-criteria user/service system modeling

To model the concept of multicriteria phenomena, we introduce different kinds of pheromones. Each kind of pheromone is represented by a specific color. We introduce the notion of center, which is a specific spatial location. On each center, we are able to define many queens. Each queen, belonging to a center, is able to emit its own pheromone which is represented by a colored pheromone different from the other queens belonging to the same center. A queen, associated to a spatial center, describes a specific criterium linked to a colored pheromone. To represent the same criterium on different centers, we use the same colored pheromone on these different spatial locations. In order to force the ants to deposit their material only near the center, we have introduced the template function.

**Definition 2.1.** A spatial multi-criteria multicenter simulation is described by a spatial domain, S, by a set of  $n_p$  centers,  $\{P_i; 1 \le i \le n_p\}$ , and by a set of  $n_c$  colors,  $\{c_i; 1 \le j \le n_c\}$ . For each center  $P_i$ , we define a  $c_j$ -colored template function,  $\Phi_{ij}: \to \mathbb{R}$ , which gives the value of the  $c_i$  template intensity on each spatial position.

For each center  $P_i$ , we can define a  $c_j$ -colored pheromone function,  $f_{ij}: S \to \mathbb{R}$ , which gives the value of the  $c_j$  pheromone intensity at each spatial position.

Remark 2.1. We can define the  $c_j$ -colored template function of the  $P_i$  center by the following radial exponential function,

$$\Phi_{ij}(x,y) = \alpha_{ij} \exp(-\beta_{ij}(d((x,y), (x_{P_i}, y_{P_i})) - r_{ij})^2)$$
(3)

where  $\alpha_{ij}$  is the template amplitude,  $\beta_{ij}$  is the template slope,  $(x_{P_i}, y_{P_i})$  are the  $P_i$  center coordinates.

We then define the  $c_j$ -colored pheromone function for the  $P_i$  center by a similar formula,

$$f_{ij}(x,y) = a_{ij} \exp(-b_{ij}(d((x,y), (x_{P_i}, y_{P_i})) - r_{ij})^2)$$
(4)

where  $a_{ij}$  is the pheromone amplitude,  $b_{ij}$  is the pheromone slope.

We then give some definitions which allow to generalize the ant nest building algorithm for the multi-criteria multi-center simulation.

**Definition 2.2.** A center  $P_i$  has the **dominant** color  $c_j$  if

$$a_{ij} = \max\{a_{ik}; 1 \le k \le n_c\}$$

**Definition 2.3.** On each space location Z = (x, y), we define the  $c_j$  colored pheromone intensity as the function  $F_j(Z)$  or  $F_j(x, y)$  defined by the formula,

$$F_j(Z) = F_j(x, y) = \sum_{i=1}^{n_p} f_{ij}(x, y).$$
 (5)

The multi-criteria multi-center model proposed here implements some spatial objects that are the material and spatial agents (which are the ants). The ants have to carry the material in order to achieve the spatial self-organization simulation.

**Definition 2.4.** A material involved in a spatial multi-criteria multi-center simulation has to include a **characteristic color table** which corresponds to the only colors that the material is able to perceive and upon which it will be able to react.

*Remark 2.2.* An ant involved in a spatial multicriteria multi-center simulation and which is carrying a material has to include a **characteristic color table** which corresponds to the material characteristic color table.

Each ant of the simulation which is carrying some material  $M_l$ , has to implement a decision process which gives, as output, a color pheromone template  $c_j$  that is used for the material transportation by the ant. This selected color  $c_j$  is called **the ant behavior**.

At each simulation step, a carried material  $M_l$ is associated to a color  $c_j$ , called the ant behavior in Definition 2.2. The ant which is carrying this material will then move by searching in its neighboring position, the appropriate one. Ant move computation is based on a specific ranking process which evaluates the greatest ranking place within the neighboring places, corresponding to the highest pheromone color rate.

**Definition 2.5.** For each material M (or the ant carrying it), we define the  $c_j$  color attribute preference as the rate, a real number  $s_{jM} \in [0, 1]$ . For each material M (or the ant carrying it) and each space location Z = (x, y), we compute the ranking,  $\rho_{MZ}$  by the formula,

$$\rho_{MZ} = \sum_{j=1}^{n_c} s_{jM} \cdot F_j(Z) \tag{6}$$

where  $n_c$  is the number of pheromone colors,  $F_j(Z)$ is the  $c_j$  colored pheromone intensity on the location Z, defined in Definition 2.3 and  $s_{jM}$  is the  $c_j$  color attribute preference for the material M defined in Definition 2.5. The ant move process consists of the moving of the ant to the place with the best ranking value.

#### 2.2.3. Experimental output and analysis

We study an experimental configuration, composed of seven centers and with random initial positions for the materials and for the ants. On each center, we put eight queens, each one is associated to a colored pheromone labeled from 0 to 7. In Fig. 4, we show the result of one simulation in which ants progressively aggregate the material around centers, following pheromone trails and based, not only on the ant move process described in the previous definition, but also on the probability of taking or depositing material defined as in Eqs. (1) and (2).

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Fig. 4. Simulation computation, at successive steps: iterations 0, 152, 250, 370, 601, 1601. For each of the seven centers, eight queens emit pheromons able to progressively aggregate materials around them, based on multi-criteria aspects.

On the left top subfigure, we see the initial distribution of materials and ants. In the five other subfigures, we see three successive steps of the simulation. We observe the formation of material affectation to each center in order to respect the attraction process, according to the material characteristics. In Fig. 5, we take a zoom of the last step of the simulation shown in Fig. 4, removing the ant representation.

## Attraction analysis based on dominant component

We need to exhibit some analysis to better understand how the computation produced, by selforganization, the distribution of the material over the whole domain. The analysis is based on the dominant component for the material: a characteristic color table is associated to each material. The process of attraction is led by the ant behavior defined previously. This ant behavior consists here of extracting from the characteristic color table, a selected color corresponding to the colored pheromone, which controls the ant in order to move to the places of highest values for this colored pheromone.

To better understand the mechanism of attraction, we have to focus on the selected color associated to each material which controls the ant displacement and to forget the other colors belonging to the characteristic color table of the material. In this part, we only associate to each material the dominant color and we study how these colored material are distributed over the entire domain. To analyze this distribution, we represent three graphs for each center. On each graph, we have a specific representation of the distribution of all attracted material according to its dominant selected color.



Fig. 5. Simulation: zoom on final step, at iteration 1601.

Before defining all these graphs, we have to define a zone of analysis for each center:

**Definition 2.6.** For each center  $P_i$  and each color pheromone  $c_j$  corresponding to the template function  $f_{ij}$  defined in Remark 2.1, we define the **referential disk** as,

$$D_{ij} = \{M = (x, y); f_{ij}(M) < ra_{ij}\}$$

where  $a_{ij}$  is the template amplitude of function  $f_{ij}$ and  $r \in [0, 1]$  is a real number, the value of which is generally equal to 0.5 in the following.

According to this referential disk, we compute, using three different methods, some indicators corresponding to the quantity of material of each color in this disk, another corresponding to some relative quantity of material in function of the pheromone amplitude and finally, the last indicator depends on the neighborhood. The three graphs used in our study are defined by,

• The material density of dominant color  $c_j$  for the center  $P_i$  which is computed as follows,

$$\rho(P_i, c_j) = \frac{\eta(D_{ij})}{\mathcal{A}(D_{ij})}$$

where  $\eta(D_{ij})$  is the number of materials of dominant color  $c_j$  inside the referential disk  $D_{ij}$  of center  $P_i$  and  $\mathcal{A}(D_{ij})$  is the area (e.g. number of material places) of the disk  $D_{ij}$ .

• The **pheromone efficiency** of dominant color  $c_i$  for center  $P_i$ , which is computed as follows,

$$\rho_r(P_i, c_j) = \frac{\rho(P_i, c_j)}{a_{ij}}$$

• The relative pheromone efficiency graph which computes the queen efficiency relatively to the neigborhood network. This computation consists of changing the pheromone amplitude used in the previous graph by a relative pheromone amplitude  $a_{ij}^r$  defined by,

$$a_{ij}^r = \frac{a_{ij}}{\sum_{k \in \vartheta_i} a_{kj}}$$

where  $\vartheta_i$  is the set of centers belonging to the neighborhood of the center  $c_i$ . The relative pheromone efficiency of dominant color  $c_j$  for the center  $P_i$  is computed as follows,

$$\rho_{sr}(P_i, c_j) = \frac{\rho(P_i, c_j)}{a_{ij}^r}$$

The last graph exhibits a complex indicator which takes into account the interaction network of the center system. In Fig. 6, we represent, for center 4, the three graphs previously defined and we represent two additional graphs corresponding to the pheromone amplitude and to the relative pheromone amplitude according to the neighborhood, for each color.

#### Results analysis

This attraction analysis with the three associated graphs, allows us to better understand the complexity of the phenomena according to the multi-criteria and to the spatial effects. To illustrate this analysis, we observe the following results of center 4, based on Fig. 6:

(1) A first remark concerns the nonlinear properties of the attraction phenomenon, which makes the color of the more important pheromone intensities attract a great number of materials of this color and a few number of materials of color of lower pheromone intensities. There is no linear relation between the number of colored material and the corresponding colored pheromone intensity. Finally, the material of color of lower

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Fig. 6. Attraction analysis for center 4: (1) material density, (2) pheromone efficiency, (3) relative pheromone efficiency, (4) pheromone amplitudes and (5) relative pheromone amplitudes.

pheromone intensity is not significant. Concerning center 4 of our current experiment, we only focus on the material of the two dominant colors: orange (color number 6) and blue (color number 1).

- (2) On center 4, the first graph material density — shows that the more attractive colored materials are, in order, the orange colored materials and then the blue colored materials. The predominance of the orange colored material over the blue is corrected on the second graph, which consists in dividing the colored material number of each color by the corresponding colored pheromone intensity. As orange pheromone is greater than the blue one, we could expect that this center will attract more orange materials than blue ones. The pheromone efficiency graph shows this, making the orange and blue curves come closer.
- (3) The second graph which is the pheromone efficiency graph, only takes into account local information about the center and not spatial

information. With the third graph which is the spatial pheromone efficiency graph, we correct the importance of the pheromone intensity of each color on a specific center with respect to the same color pheromone intensity of the neighboring centers. In Fig. 5, we observe that the neighbors of center 4 are centers 1, 3 and 5. For these three centers, the blue pheromone intensity is low. Moreover the orange intensity is high at center 5. Therefore, if we observe part (5) of Fig. 6, we can see that the relative orange intensity becomes lower than the relative blue pheromone intensity. The last graph — relative pheromone efficiency — shows a correction according to this relative pheromone intensity. But, finally, this graph shows that the orange material number is still greater than the blue one. That is an unpredictable event.

Of course, the complexity of simulation is not completely predictable by nature and unpredictable phenomena appear as we finally observe in the previous example. These phenomena are mainly due to the complexity of spatial configuration and multicriteria characteristics. These unpredictable characteristics of the result overtop our advanced analysis which integrates a spatial interaction correction of first order (e.g. only direct neighbors are considered in this analysis).

#### 2.3. Application to urban dynamics

The impressive increase of economical, technological, social and environmental changes in our world makes their management become a great challenge. Implementing wrong policies to solve such problems can make it become worse or can generate new ones. Many of the problems we have to face nowadays arise as nonpredictable side effects of our own past actions. Most of these wrong policies are not flexible or adaptive and we cannot change them in order to achieve our goals within a dynamic solution environment. In other words, solving such problems by studying a part of them without modeling the complexity of the different parts can provide an efficient solution for short term but a negative one for long term, leading to nonreversible system evolution.

Effective decision making and learning in a world of growing dynamic complexity requires us to search for new ways of system modeling and to expand the boundaries of our mental models. We have to develop tools in order to understand how the structure of complex systems creates their own behavior. Our purpose is to analyze organizations or societies within their spatial complexity. Solving this problem in sustainable way, should start by understanding how the individual behavior in the organization affects the whole system behavior. Observing complex self-organized systems in nature (like social insects) and understanding them leads to discovering concepts of emergence [Bertelle et al., 2009. The power of these systems does not come from any central control but from their flexible interactions with themselves and with their environment in an adaptive way. This problem involves complex networks of location interactions, complex networks of individual characteristics interaction and even complex networks of multi-scale decision making, that are the decision of individuals, the decision of services managers and the decision of society development planners.

The previous model has been applied to cultural urban dynamics model in order to understand how citizens are using a set of cultural centers according to the spatial configuration of this system. For this application, a queen describes a cultural center which emits attraction function. A material represents a potential user whose characteristics like age, gender, social level or educational level, are represented by the characteristic color table [Ghnemat, 2009].

The morphogenesis of complex systems, presented here, results from the decentralized process of self-organization. Bioinspired models used in these simulations, can be easily adapted to various phenomena where spatial structural organization development is essential in the process. Control of the system building is expressed globaly with template functions, but each elementary entity behavior has high-level of freedom degree and is globaly controlled by the self-organization mecanism. In the following section, we study another stage of system complexity evolution: the system is built and based on structures which are described by complex networks. The objective is then to model and study the control of such complex structures.

## 3. Dynamical Control and Synchronization of Complex Networks — Concepts and Application

In this section we are interested in the dynamics of complex systems. We consider that the structure of the system is already established by the process of morphogenesis described in the previous section. We study how this structure influences the evolution of the system and the emergence of global properties. The behavior of each entity is described by a dynamical system and the interactions between the entities form a complex network. An important property characterizing complex systems is that they have less degrees of freedom than the degrees of freedom of their components summed up. It is important to know the influence of different parameters on the system in order to control it. Here, we show how a particular parameter, the interaction strength between the entities, can be used in order to synchronize a network and how this parameter should be adjusted for networks of different topologies.

We are particularly interested in the emergence of synchronization of the entities constituting the system, a phenomenon of major importance in many disciplines. Synchronization motion of dynamical systems is of main interest in different scientific domains. During the last two decades, synchronization has attracted a great deal of attention [Ott et al., 1990; Chen et al., 1998; Derivière & Aziz-Alaoui, 2003; Aziz-Alaoui, 2006] since the seminal papers by Yamada and Fujisaka [1983] and Pecora and Carrol [1990]. Let us recall that synchronization is a phenomenon characteristic of many processes in natural systems and nonlinear science. It remains the subject of intensive research and is today considered as one of the basic nonlinear phenomena studied in mathematics, physics, engineering or life science. This word has a Greek root, syn = common and chronos = time, which means to share common time or to occur at the same time, that is correlation or agreement in time of different processes.

Synchronization of two dynamical systems generally means that one system somehow follows the motion of another. A lot of research has been carried out and, as a result, showed that even chaotic systems could synchronize when they are coupled. Many researchers have discussed the theory, the design or applications of synchronized motion in coupled chaotic systems.

There are different synchronization regimes, some of which are: identical (or complete) synchronization, which is defined as the coincidence of states of interacting systems; generalized synchronization, which extends the identical synchronization phenomenon and implies the presence of some functional relation between two coupled systems; phase synchronization, which means driving of phases of chaotic oscillators, whereas their amplitudes remain; or the burst synchronization, which means that all the oscillators fire the same number of bursts starting at the same moment.

In order to show the influence of the network structure on the different types of synchronization, we will use a model from the domain of neuroscience, described in the next section.

#### 3.1. Hindmarsh-Rose model

In 1952, a mathematical model that describes neuron activity has been proposed by two neurophysiologists, Hodgkin and Huxley [1952]. Different neuron models have been then developed and studied, see

for example [Izhikevich, 2007; Morris & Lecar, 1981] and references therein cited. In this paper, we consider one of them, the Hindmarsh–Rose model (HR), which results from a simplification and a generalization of the Hodgkin–Huxley model, see [Hindmarsh & Rose, 1982, 1984]. As observed in various biological systems, neuron activity presents different time scales. This can be explicitly observed in the HR model, which is a *slow-fast* autonomous system of three ordinary differential equations. The two first equations control the fast dynamics while the third one controls the slow dynamics. The HR model reads as follows,

(HR) 
$$\begin{cases} \dot{x} = y + ax^2 - x^3 - z + I \\ \dot{y} = 1 - dx^2 - y \\ \dot{z} = \epsilon (b(x - c_x) - z) \end{cases}$$
(7)

Parameters a, b and d are experimentally determined,  $c_x$  is the equilibrium x-coordinate of the two-dimensional system given by the first two equations of (7) when I = 0 and z = 0 and parameter Icorresponds to the applied current. Finally, parameter  $\epsilon$  represents the ratio of time scales between fast and slow fluxes across the membrane of a neuron. This HR neuron model can exhibit most of biological neuron behavior, such as *spiking* or *bursting*. With appropriate parameter settings, the HR model exhibits periodic behavior characterized by fast periods of spiking called bursts, followed by slow quiescent inter-burst periods, as shown in Fig. 7.

Hereafter, for all numerical experiments, we use HR system with the following coordinate changes, see [Belykh *et al.*, 2005], y = 1 - y, z = 1 + I + z,  $d = a + \alpha, c = -1 - I - bx_c$ . Applying this transformation, we obtain,

$$\begin{cases} \dot{x} = ax^2 - x^3 - y - z \\ \dot{y} = (a + \alpha)x^2 - y \\ \dot{z} = \epsilon(bx + c - z) \end{cases}$$
(8)



Fig. 7. Time series (t, x) of (8) when parameters are fixed as in (11). For this set of parameter values, a HR system exhibits a periodic bursting behavior.

Nerve cells interact via synapses. A synapse is the functional contact part which exists between two neurons or between a neuron and another cell (muscular cell, sensory receptor, etc.). It operates the conversion of an action potential fired by the presynaptic neuron into a signal in the postsynaptic cell. Usually, two different types of synapses are discerned. Chemical synapses need some neurotransmitters to transmit information while through electrical synapses, the signal is transmitted electrically through gap-junctions. The size of the synaptic cleft is characteristic of one or the other kind of synapses. In the case of electrical synapses, this synaptic cleft is about two nanometers, while it can reach from ten to forty nanometers in the case of chemical synapses. Since the large majority of synapses in the nervous system are chemical, we focus our study on them. They can be modeled by nonlinear coupling functions.

Let us consider a network composed by n HR neurons. These neurons are coupled by their first variable  $x_i$ . A model of this network is given by,

$$\begin{cases} \dot{x_i} = ax_i^2 - x_i^3 + y_i - z_i - \sum_{j=1}^n c_{ij}h(x_i, x_j) \\ \dot{y_i} = (a + \alpha)x_i^2 - y_i \\ \dot{z_i} = \epsilon(bx_i + c - z_i) \end{cases}$$
(9)

for i = 1, ..., n, where h is the coupling function and  $\{c_{ij}\}$  is the network adjacency matrix. When the neurons are chemically coupled, the coupling function h is given by [Belykh *et al.*, 2005] and reads as, Modeling the Dynamics of Complex Interaction Systems

$$h(x_i, x_j) = g_{\text{syn}} \frac{(x_i - V)}{1 + \exp(-\lambda(x_j - \Theta))}$$
(10)

where  $g_{\text{syn}}$  is the coupling strength,  $\Theta$  is the threshold reached by every action potential for a neuron. Parameter V is the reverse potential and must be larger than  $x_i(t)$  for all i and all t since synapses are supposed excitatory. Throughout this paper, parameters are fixed as follows,

$$a = 2.8, \quad \alpha = 1.6, \quad c = 5,$$
  
 $b = 9, \quad \epsilon = 0.001$  (11)

$$V = 2, \quad \lambda = 10, \quad \Theta = -0.25$$
 (12)

The parameter  $g_{\text{syn}}$ , the synaptic coupling strength is not fixed and we study its influence on the global network behavior as a function of the network topology in the remainder of this section.

## 3.2. Complete synchronization and associated constraints on the network topology

We first focus on the complete synchronization phenomenon within networks defined as in (9) with the coupling function given in (10). In [Belykh *et al.*, 2005], it has been shown that the complete synchronization phenomenon can only arise when all the neurons of the network receive the same number of signals from the other neurons. This condition seriously limits the possible network topologies. For different networks satisfying this necessary condition, such as rings, complete networks or k-regular networks (see Fig. 8) both oriented and non-oriented,



Fig. 8. (a) *Ring topology network*: each neuron is connected to its two closest neighbors. (b) *Full topology network*: each neuron is connected to all the others. (c) *k-regular topology network*: each neuron is connected to its k nearest left and right neighbors.

we consider the minimal coupling strength needed to obtain complete synchronization.

Our study points out that this coupling strength follows a law given by Eq. (13).

$$g_{\rm syn}^d = \frac{g_{\rm syn}^\star}{d} \tag{13}$$

where  $g_{\text{syn}}^{\star}$  is the coupling strength needed to make two bidirectionally coupled neurons synchronize and *d* is the in-degree of the neurons (see also [Corson & Aziz-Alaoui, 2009]). This law presents the classical shape of power laws, often observed in complex systems, as one can see in Fig. 9. This law is followed whatever the network topology is, as far as all the nodes have the same in-degree.

The simple expression of the coupling strength given by (13) is possible only because the necessarv condition for complete synchronization leads to quite simple and regular topologies. These topologies are unrealistic because as we have seen in the previous section, the morphogenesis process leads to complex interaction patterns and networks with specific degree distributions and other properties. That is why in the subsequent sections we are interested in a weaker synchronization phenomenon, called burst synchronization. The conditions on the network to make burst synchronization appear are weaker than the ones needed to observe a complete synchronization phenomena [Batista et al., 2010; Han et al., 2009: Shi & Lu, 2009: Wang et al., 2007; Zheng & Lu, 2008]. On the other hand, burst



Fig. 9. Observed and predicted coupling strength for complete synchronization in a full network as a function of the network size. The results are similar for the other network topologies studied.

synchronization is a more subtle phenomenon that cannot be described by simple laws as (13).

## 3.3. Detection of burst synchronization in complex networks

We consider that a network of coupled neurons presents a burst synchronization behavior if the neurons fire bursts start all at the same time. Unlike complete synchronization, burst synchronization is not easy to detect numerically. Even the distinction between fast and slow periods could be difficult, since the dynamics of single neurons could change in unpredictable way when the coupling force is slightly modified.

In this section we propose a general algorithm of burst synchronization detection in networks of coupled oscillators. Its application is not restricted to HR neurons, it can be used to detect burst synchronization in networks composed of any oscillators displaying burst behavior.

Our algorithm can be decomposed in four main steps. In order to detect burst synchronization, bursts of different neurons must be matched. To do this, one needs to determine the start time of each burst, and before detecting bursts, spikes must be detected first.

#### 3.3.1. Spike detection

The first step of our algorithm is the detection of spikes. For our needs it suffices to find the local maxima of  $x_i(t)$  for each neuron *i*. Thus each spike is associated to the time when the corresponding local maximum occurs. Figure 10 shows an example of time pattern of bursts in a network of five neurons. Each spike is represented as a point on the horizontal line corresponding to the neuron in which the spike occurs.

#### 3.3.2. Burst start detection

Once all spikes are localized, we need to determine the first spike of each burst. Since each burst is preceded by a quiescent period, the idea is to consider the inter-spike distances. When the distance between two consecutive spikes is large enough, the second spike is considered as the first of a new burst. It is not easy to determine a threshold beyond which an inter-spike distance could be considered as "large enough". Indeed, the coupling force applied and the

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Fig. 10. Illustration of the burst synchronization detection algorithm. The four steps of the algorithm are represented in this figure.

in-degrees of the network nodes change significantly the form of the attractors of individual neurons and a threshold applicable to a given combination of these parameters could be completely inappropriate for another combination. To overcome this difficulty, we use the k-means clustering method [Macqueen, 1967] to separate the inter-spike distances in two groups. When applied to classify one-dimensional observations into two groups, the k-means clustering method is very simple. Suppose that  $d_1, \ldots, d_n$  are the distances between all pairs of consecutive spikes of a given neuron sorted in non-decreasing order. For fixed i, let  $\mu_1(i)$  be the mean of  $d_1, \ldots, d_n$  and let  $\mu_2(i)$  be the mean of  $d_{i+1}, \ldots, d_n$ . Let

$$i^{\star} = \underset{1 \le i \le n}{\operatorname{argmin}} \left\{ \sum_{j=1}^{i} (d_j - \mu_1(i))^2 + \sum_{j=i+1}^{n} (d_j - \mu_2(i))^2 \right\}$$

Then  $d_{i^*+1}, \ldots, d_n$  are considered as inter-burst distances. Figure 10 shows the bursts detected by our algorithm as segments below the spike patterns.

For certain coupling force values and network topologies, the behavior of individual neurons can change from bursting to spiking. An indicator allowing to distinguish between these two behaviors is the ratio between the smallest inter-burst distance and the largest inter-spike distance,  $\frac{d_{i^*+1}}{d_{i^*}}$ . This ratio close to one indicates spiking behavior. With our parameter settings, this ratio is about 4.2 for a single noncoupled neuron.

#### 3.3.3. Burst matching

The next step of our algorithm is to match the bursts fired by different neurons. We define the distance between two bursts as the absolute value of the difference of their starting times determined at the previous step. Let  $b_1$  be a burst emitted by a neuron and  $b_2$  be a burst emitted by another one. We say that  $b_1$  and  $b_2$  match if  $b_1$  is the burst of the first neuron closest to  $b_2$  and  $b_2$  is the burst of the second neuron closest to  $b_1$ . We say that bursts  $b_1, \ldots, b_n$  fired by *n* different neurons match if each pair of them match. Figure 10 represents groups of matching bursts by dots at the beginning of the burst segments.

A necessary condition for burst synchronization is that almost all bursts belong to *n*-tuples of matching bursts. This condition is measured by the ratio of the number of bursts belonging to matching *n*tuples and the total number of bursts in the network, over a long time period.

#### 3.3.4. Burst synchronization

Another condition for burst synchronization is that all the bursts within a matching group start in a small time interval. We measure the largest distance between two bursts in each *n*-tuple of matching bursts and we take the mean of this distance over all *n*-tuples in the observed period. The distances are visualized by grey vertical lines in Fig. 10. If this average is below a given threshold, then the oscillators are said to exhibit burst synchronization. The thresholds used in the numerical experiments are discussed in the next section.

To recapitulate, there are three conditions for burst synchronization:

- The oscillators must have bursting behavior, or the ratio  $\frac{d_{i^*+1}}{d_{i^*}}$  must be above a given threshold for all oscillators in the network.
- The ratio between the number of matching bursts and the total number of bursts must be close to one.
- The average span of the bursts within each matching group must be below a given threshold.

The algorithm presented in this section is applied on networks of different topologies. The results are discussed in the next section.

## 3.4. Burst synchronization in complex network of different topologies

In this section, we study how the network structure influences the values of the synaptic coupling strength  $g_{\rm syn}$  for which burst synchronization appears. The numerical results are obtained using a Java code implementing the Runge–Kutta 4 integration method. GraphStream library [Dutot *et al.*, 2007] is used for generation and manipulation of networks. First of all, let us see on what types of network topology we can expect burst synchronization.

A necessary condition for synchronization of two network nodes is that either one of them must be influenced by the other or both of them must be influenced by a third node. At network level this implies the existence of at least one "root" node from which all nodes can be reached.

We impose a second condition, the absence of cycles in the network. The reason for this condition is that cycles could significantly modify the individual neuron behavior. In the presence of cycles, the bursting phenomenon could even disappear for certain coupling strength values. To illustrate this fact, let us consider the simplest cycle case, two neurons with bidirectional coupling. Figure 11 shows that when the coupling strength grows the bursting is progressively transformed in spiking. From the moment when complete synchronization is observed, bursting behavior comes back but in different form and disappears again for very big coupling forces. In the presence of longer cycles, the individual behavior could be even more perturbed. Our experiments show that in acyclic networks bursting is more stable.

When the network is acyclic, there exists exactly one root node which receives no signal and such that there exists a path from it to any other node. Intuitively, nodes which are closer to the root will need smaller coupling strength to synchronize with it. To check this hypothesis, we use a simple chain network in which each node receives a signal from the previous (see Fig. 12). Figure 13 shows the values of the three burst synchronization indicators defined in Sec. 3.3 as functions of the network diameter and the coupling strength. For each indicator there is clearly expressed phase transition leading to separated zones between "small" and "big" values. The boundaries between these zones are shown as green lines in Fig. 13. When we put the three boundaries together, we obtain the result shown in Fig. 14. One can see that the first indicator dominates the other two. In other words, if all network nodes emit clearly separated bursts, these bursts are synchronized. Another observation is that in the case of



Fig. 11. Times series (t, x) of a neuron (7) within a network with cycles for an increasing coupling strength (from left to right and from top to bottom) 0.1, 0.5, 1.0, 1.3 and 3.0. For certain values of the coupling strength, the bursting behavior is lost.



Fig. 13. Burst synchronization indicators as functions of the network diameter and the coupling strength for chain networks. (a) Ratio between the shortest inter-burst and the longest inter-spike distances. (b) Proportion of matching bursts. (c) Average span of burst starts within matching groups.



Fig. 14. Thresholds of the burst synchronization indicators. Burst synchronization occurs in the zone above indicator 1 (ratio between the shortest inter-burst and the longest interspike distances). The straight line "sync threshold" is a linear approximation of indicator 1.

chain network the minimal coupling force needed to obtain burst synchronization is roughly a linear function of the network diameter.

We obtain exactly the same results for tree-like topology, because in this kind of networks each path from the root to another node synchronizes exactly like a chain.

In chain and tree networks each node except the root receives a signal from exactly one node. To check if the synchronization threshold depends also on the in-degrees of the network nodes, we generate special kind of networks which we call level networks. We start from a random tree with given number of nodes and given height. Then we randomly add extra arcs only between nodes of consecutive levels. In this way we do not change the diameter of the network and keep it acyclic. Each edge added increments the in-degree of some node. We will study the influence of a parameter called density, which is the ratio between the number of added arcs and the number of possible arcs between nodes of consecutive levels. In this way a network of density 0 is a tree and in a network of density 1 all arcs between adjacent levels are present. An example of level network is given in Fig. 15.

As in the case of chain networks, the three burst synchronization indicators exhibit phase transition for level networks. Once again, the first indicator dominates the other two. The coupling strength needed to obtain burst synchronization is roughly a linear function of the network diameter, as shown



Fig. 15. Example of level networks with 50 nodes and diameter 10. Only nodes from adjacent levels are connected by arcs.



Fig. 16. Coupling strength needed for burst synchronization for level graphs of different density.

in Fig. 16. But the slope of the line depends on the density of the network. When the density increases, the slope tends to zero.

The experimental results presented in this section show that the coupling strength needed to synchronize a network of coupled oscillators depends on its structure. We have identified two network parameters influencing the synchronization threshold, the network diameter and the network density. Nodes close to the root are easier to synchronize than distant nodes. On the other hand, long distances can be compensated by high link density.

#### 4. Conclusion

In this paper, we proposed two different approaches to study complex systems. The common feature of these approaches is that they are based on structural properties. This paper aims to contribute to the formalization of complexity in dynamical organizations. These organizations are studied here in terms of structures and topologies. How does the network behavior induce modifications of the network topology? How does the topology influence the network behavior? Some elements of answers are presented based on examples from the domains of geography and biology. In the first one, emergence of organizations through a reconstruction process, is observed. In the second one, emergence of synchronization is exhibited, based on parameter control. Our purpose is to highlight that self-organization and system control are two facets of the complexity of dynamical systems. This paper contributes to propose engineering methods to manipulate such concepts which have been observed for a long time in various natural and artificial systems, difficult to formalize and understand, because of the lack of practical tools to model or analyze the emergent processes.

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