# A Generalization of the Optimal Linear Arrangement Problem

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Abstract : Given a graph, the Optimal (or minimal) Linear Arrangement (OLA) problem consists in finding a vertex ordering that minimizes the sum of edge lengths. The length of an edge depends on the number assigned at its ends in the linear ordering. We study a generalization of (OLA) in which a weight is associated to each vertex, as well as a flow to each edge. The length of an edge is define as the sum of the weights of vertices which are between the ends (in the linear ordering), times the flow on the edge.  As in (OLA), we seek to minimize the sum of edge lengths. Taking all weights and all flows equal to one gives the standard problem (OLA). Thus, this generalisation is NP-Hard as OLA is NP-Hard. The problem comes from an application in maritime transportation consisting in finding a suitable assignment of ships on a terminal quay viewed as a line. An original formulation in which an assignment is viewed as finding an optimal hamiltonian tour in a weighted graph is presented. The corresponding mathematical program is written and solved using a lagrangean relaxation approach in which the dual problem  involved as sub-problems shortest paths sub-problems and TSP sub-problems. To improve the corresponding lower bound, some valid inequalities are added to the shortest path sub-problems.  Numerical results obtained with QAPLIB benchmark are finally shown.