Comparison of Bundle and Classical Column Generation

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INRIA - Rhônes-Alpes *Université Bordeaux 1* *Université de Genève*

"Convex Optimization and Dantzig-Wolfe Decomposition"

`http://www.math.u-bordeaux.fr/MAB/ODW/`

Solve the Dantzig-Wolfe reformulation of an IP

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Original P: $$\min\{c\,x:\ A\,x\ \geq b, x \in X = \{x^i\}_i\}$$

Master IP: $$\min\{\sum_i c\,x^i\,\lambda_i : \sum_i A\,x^i\,\lambda_i \geq b,\ \sum_i \lambda_i = 1, \lambda_i \in \{0,1\}\ \forall i\}$$

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Relaxed P: $$\min\{c\,x : \; A\,x \;\geq b, x \in conv(X)\}$$

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Dual Master: $\max\{\underbrace{b\,u - r}_{\eta} : uA\,x^i - r \leq c\,x^i\;\forall i,\; u \geq 0\}$

$$\max\{\eta : \eta \leq \underbrace{c\,x^i + u\,(b - A\,x^i)}_{L(x^i, u)}\,\forall i,\; u \geq 0\}$$

Lagrangian D: $\max\{\theta(u) : \; u \geq 0\}$ with $\theta(u) := \min_i L(x^i, u)$

Standard column generation: primal view

Master Problem

$$\begin{aligned}
\text{min} \quad & \sum_{i \in I} (cx^i)\lambda_i \\
\text{s.t.} \quad & \sum_{i \in I} (Ax^i)\lambda_i \geq b \\
& \sum_{i \in I} \lambda_i = 1 \\
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\end{aligned}$$

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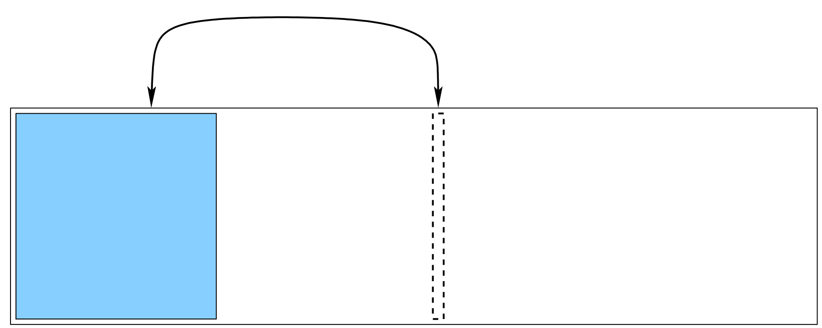
Subproblem (Oracle) : $\min_{i \in I} (c - u^k A)x^i = \min_{x \in X} (c - u^k A)x \ \rightarrow \ x^{k+1}$



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If $((c - u^k A)x^{k+1} - r^k) < 0$, add $k+1$ to I^k. **Otherwise,** STOP.

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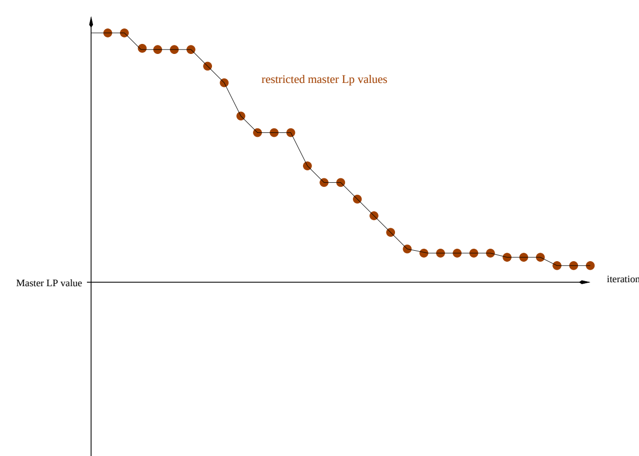
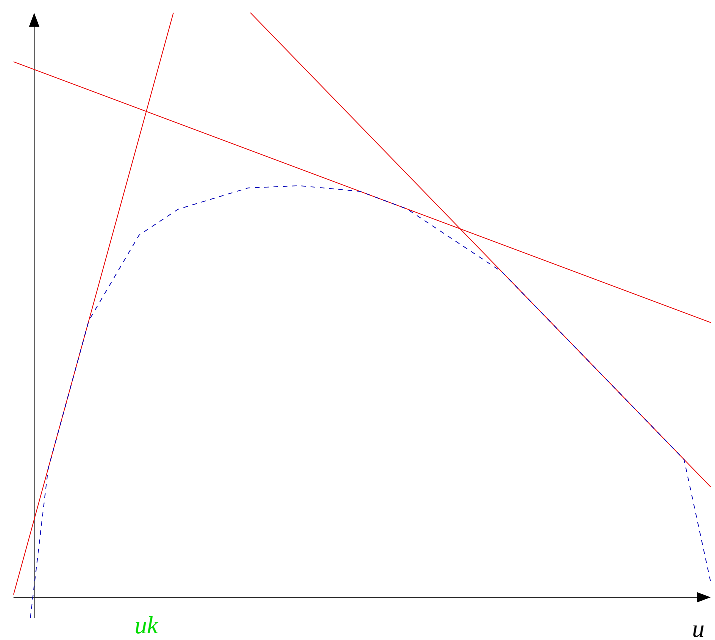


Standard column generation: dual view

Restricted Master Dual: $\max\{\eta : \eta \leq \underbrace{cx^i + u(b - A\,x^i)}_{L(x^i,u)} \,\forall i \in I^k,\ u \geq 0\}$



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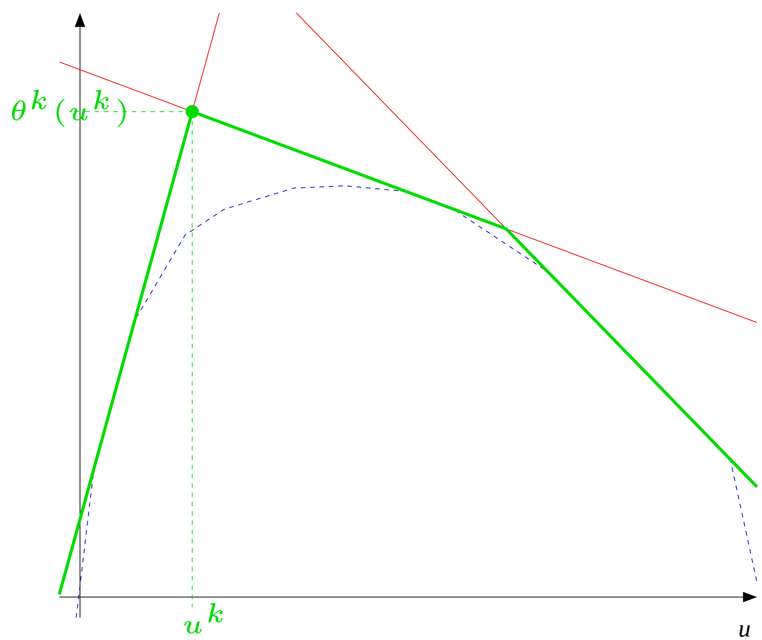

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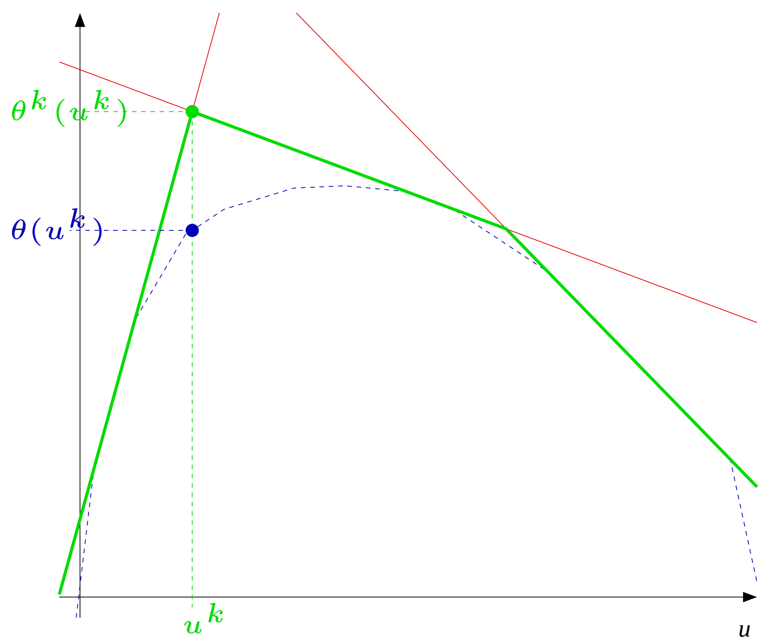

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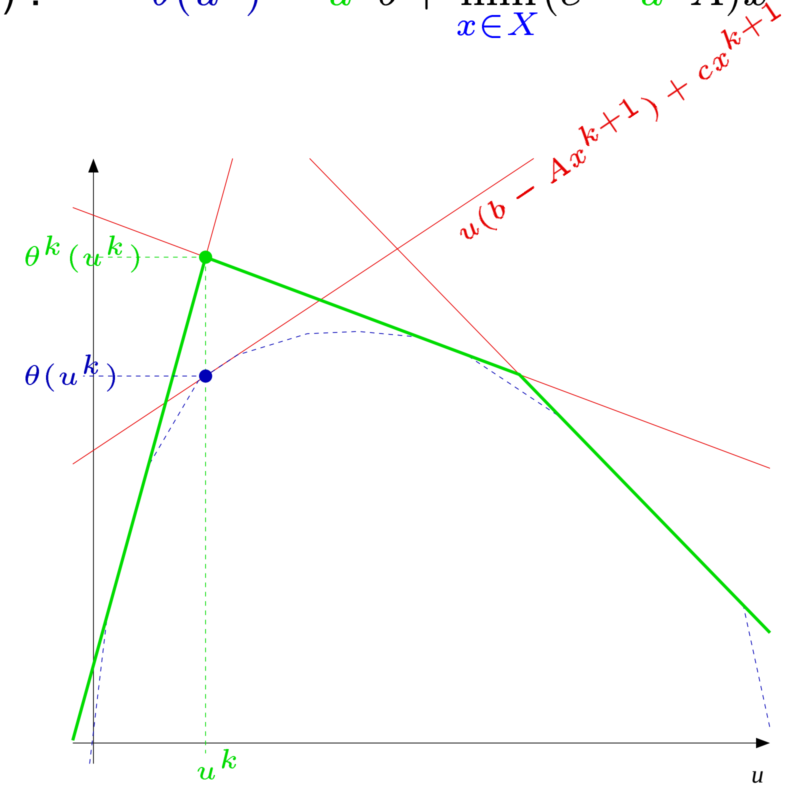

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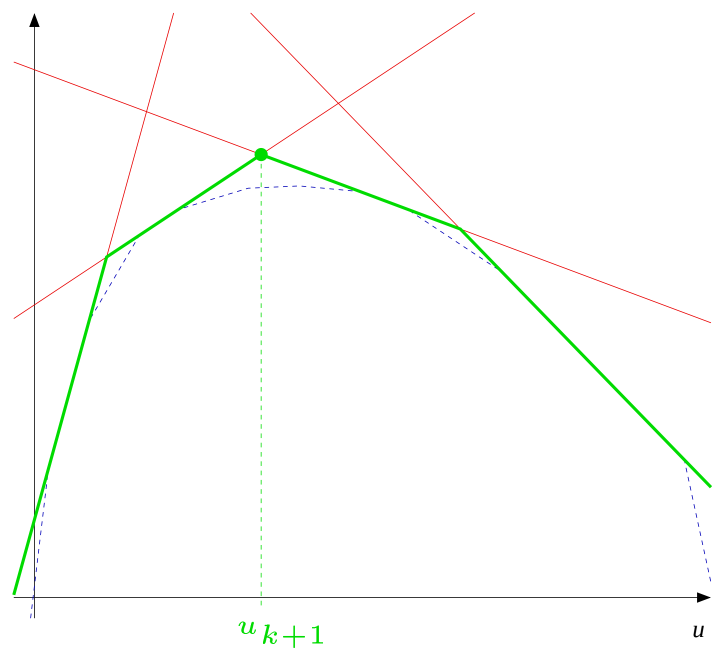

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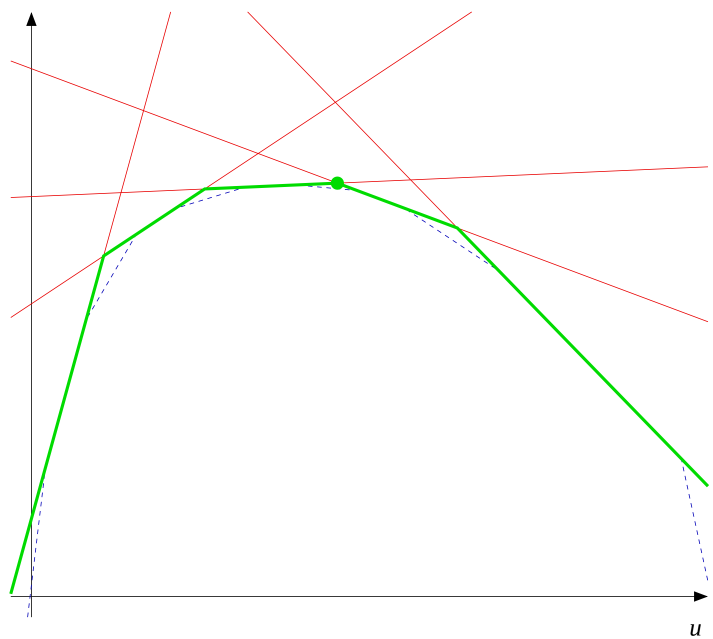

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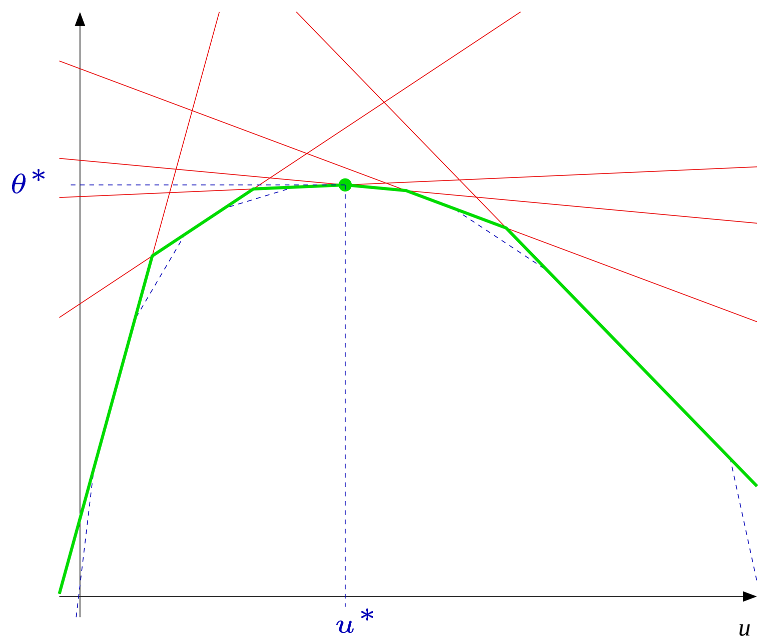


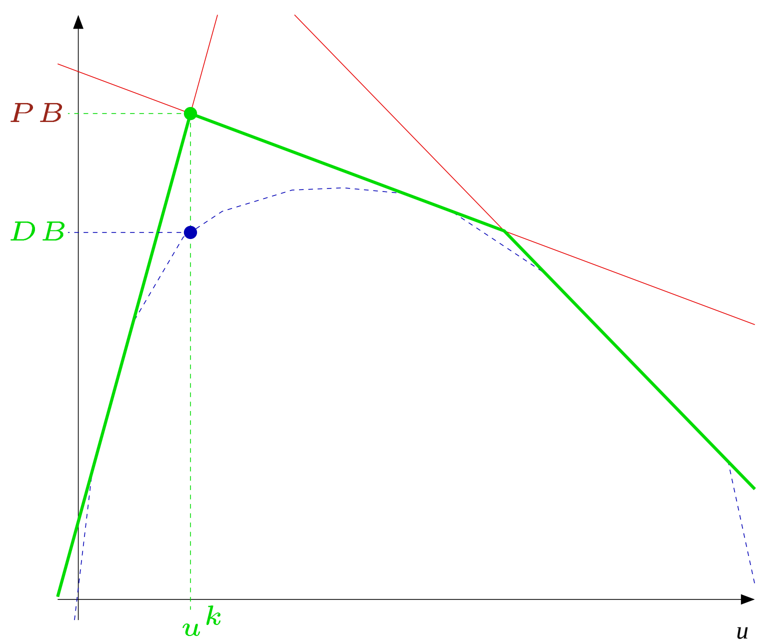
Kelley's cutting plane algorithm

Standard column generation: primal/ dual view

At iteration k, one has

A Primal Bound on the master: $PB \equiv \theta^k(u^k) = \sum_{i \in I^k} cx^i \lambda_i$

A Dual Bound on the master: $DB \equiv \theta(u^k) = PB - (c - u^k A)x^{k+1}$



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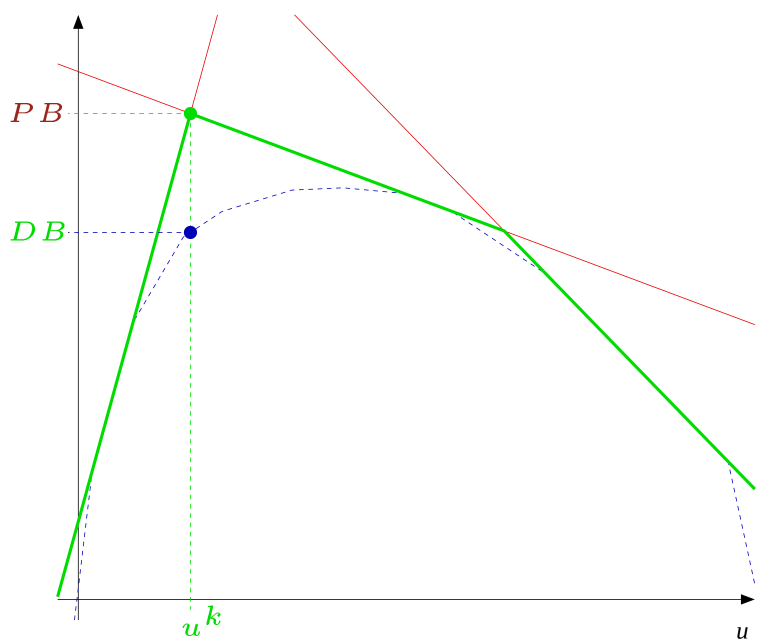
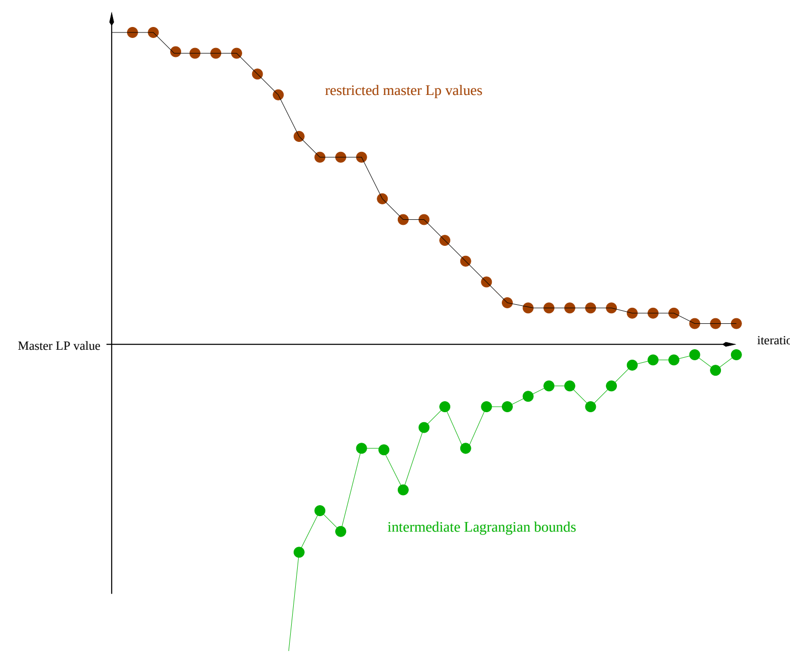
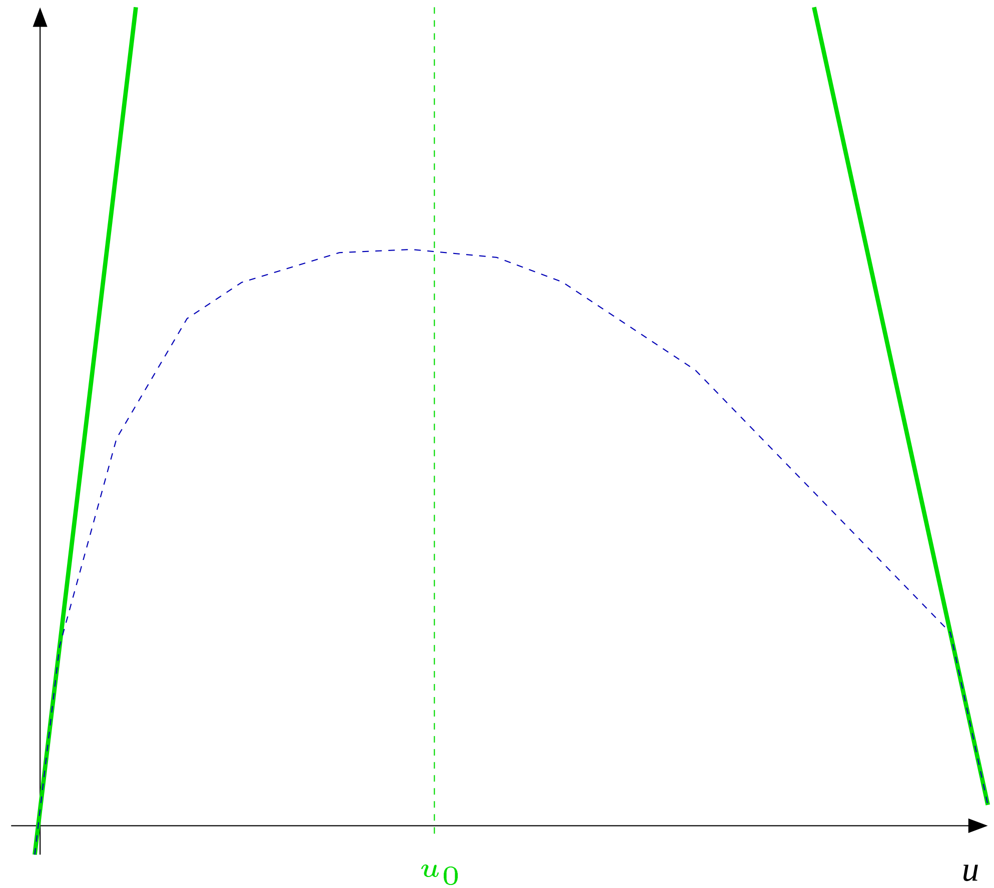




Slow convergence of Kelley's algorithm



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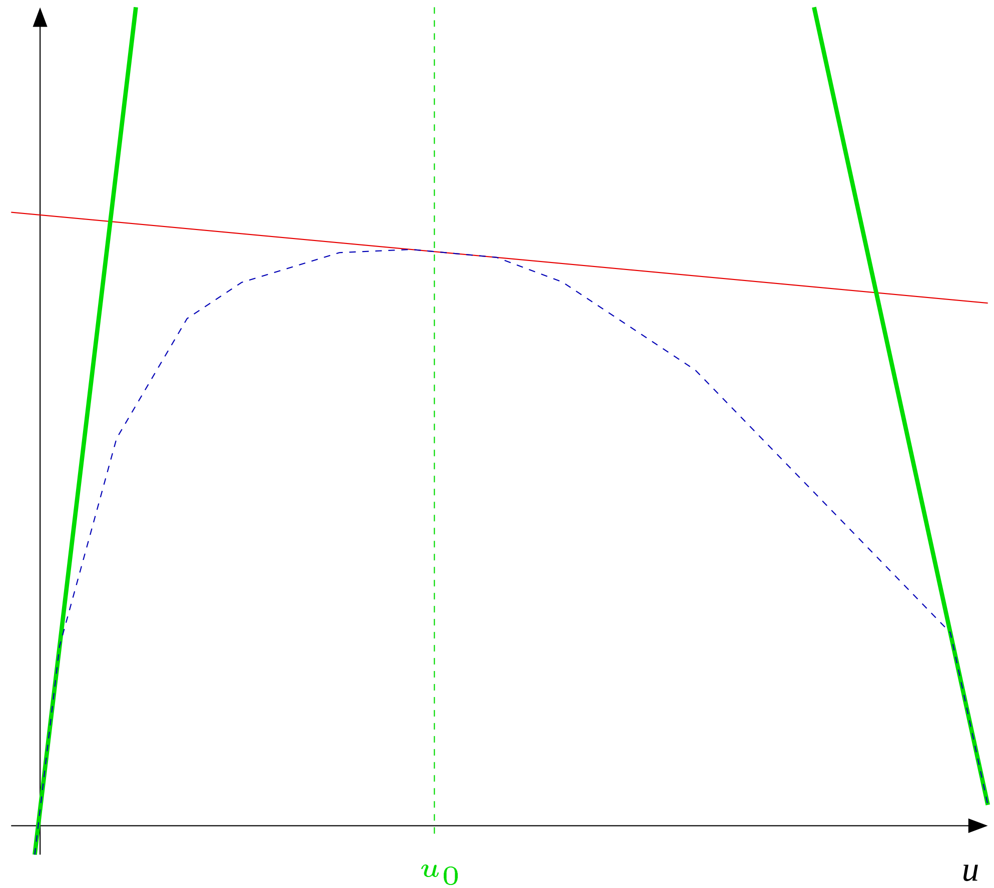

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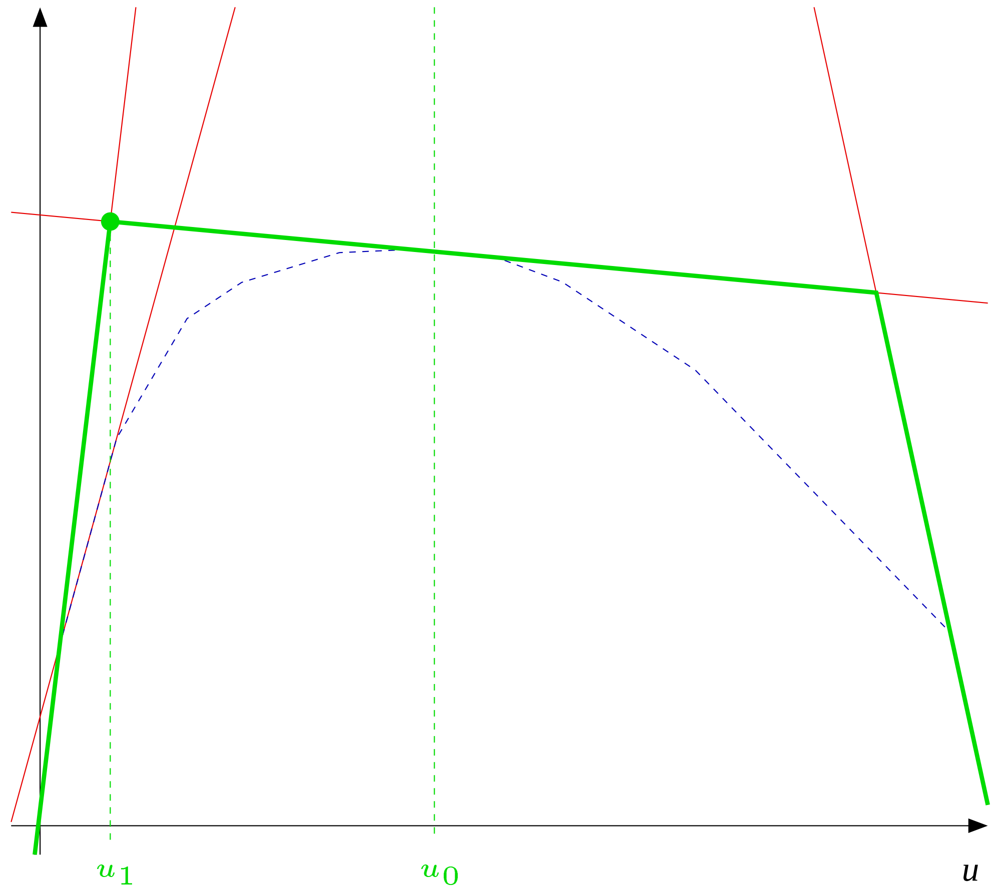

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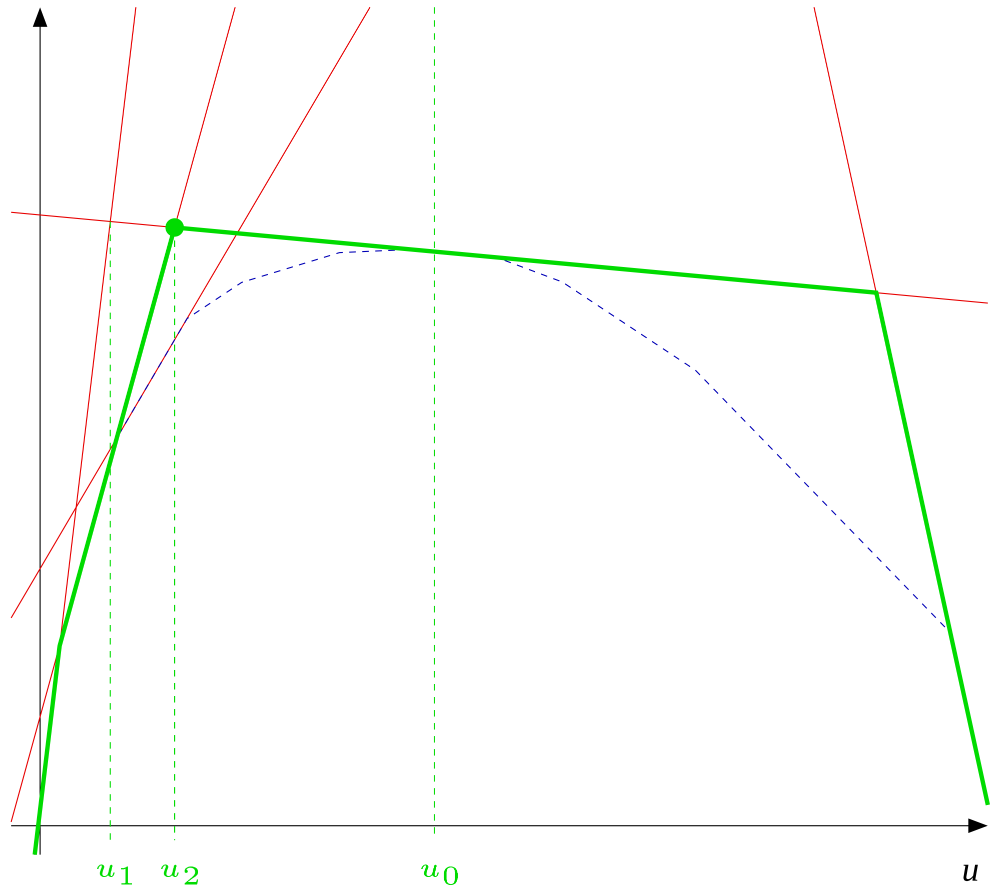

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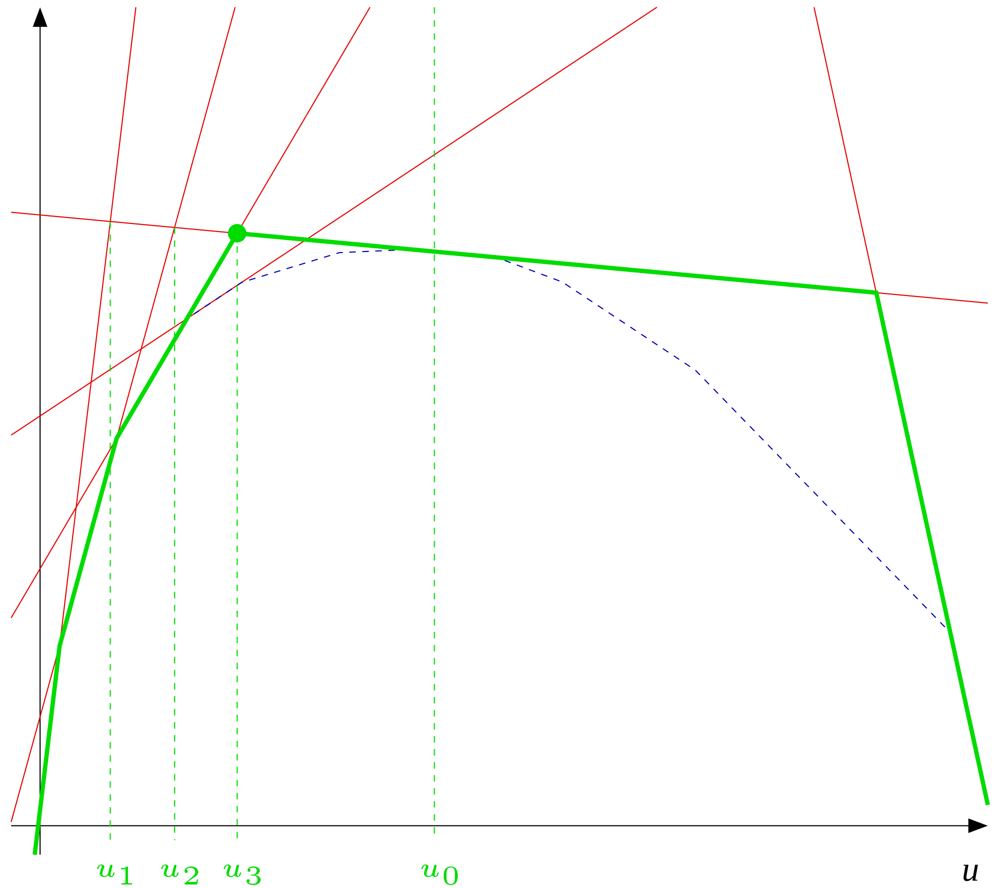

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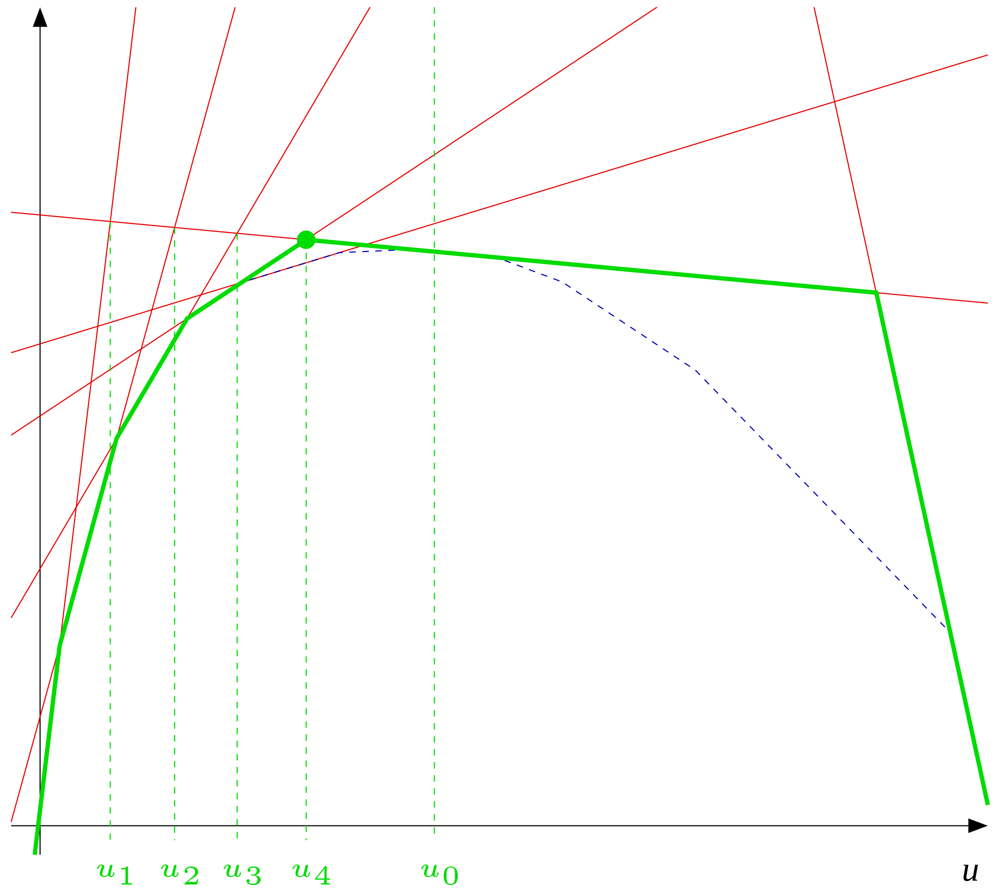

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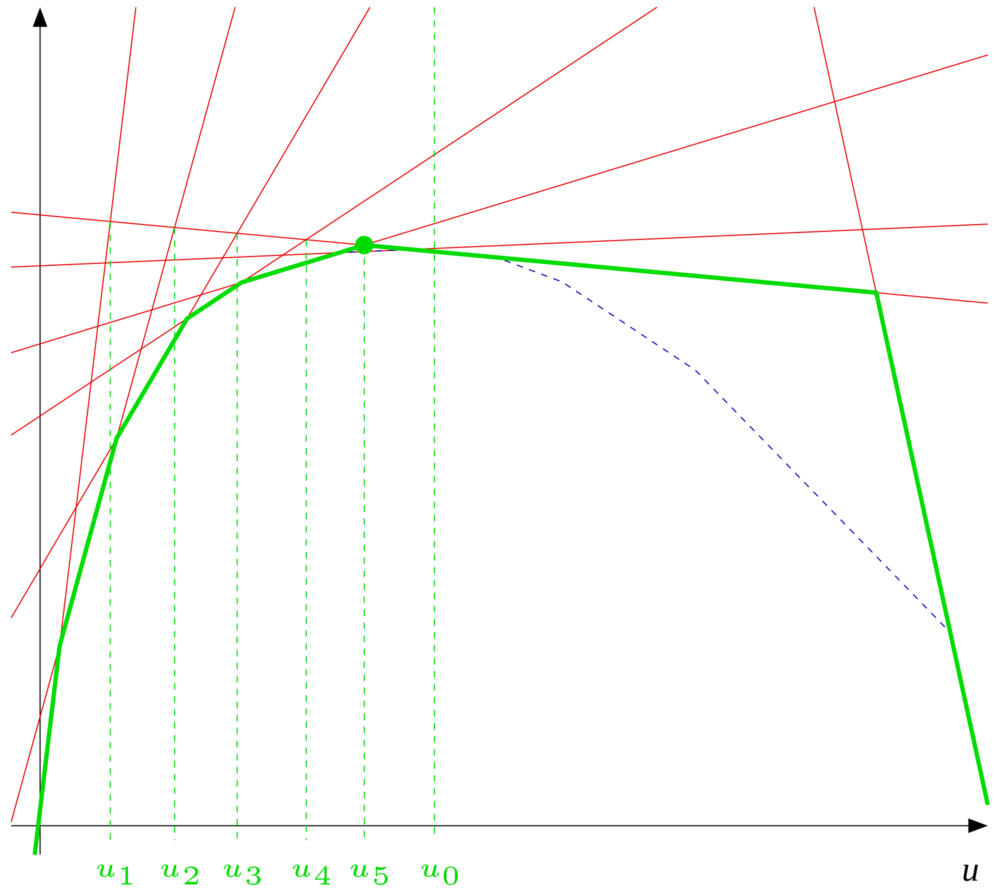

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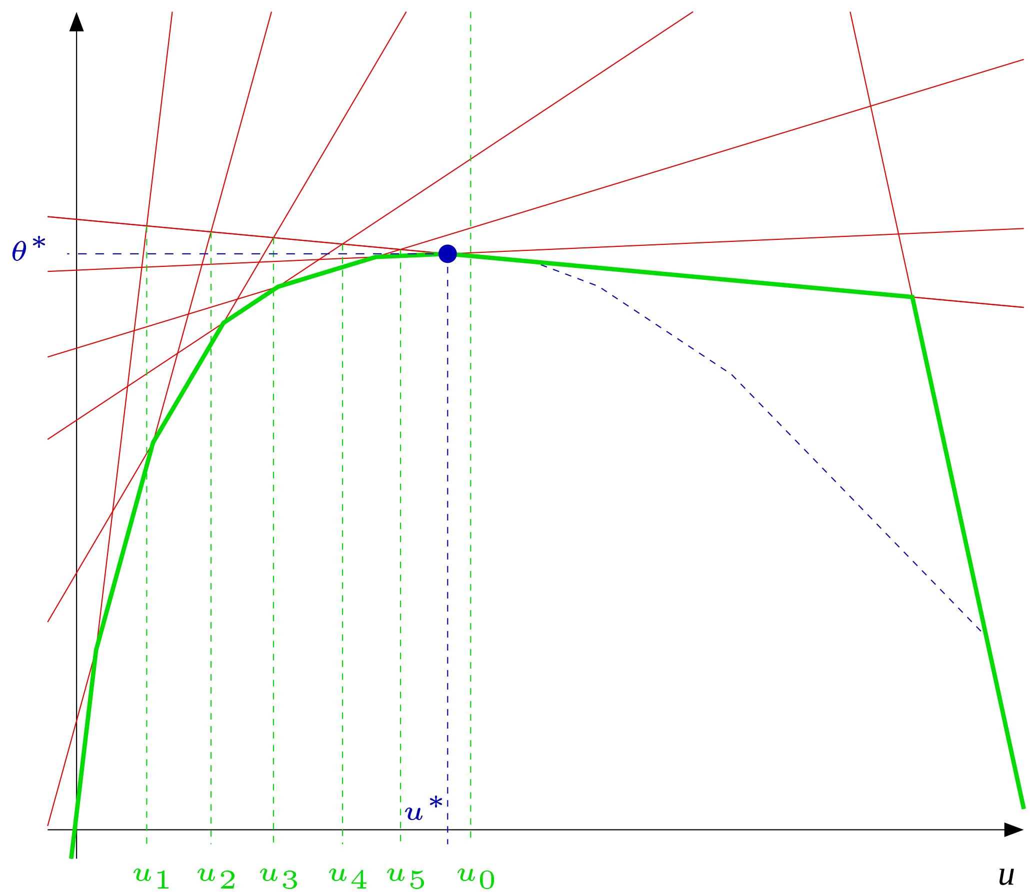
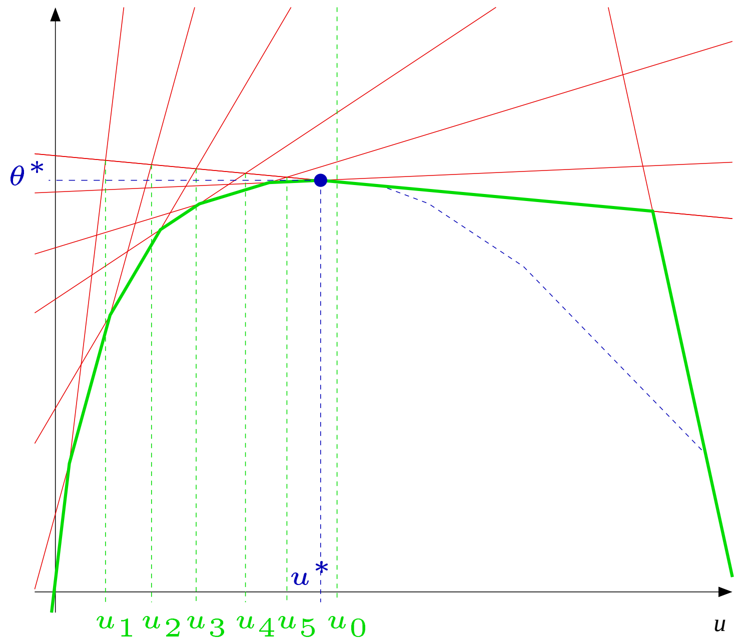
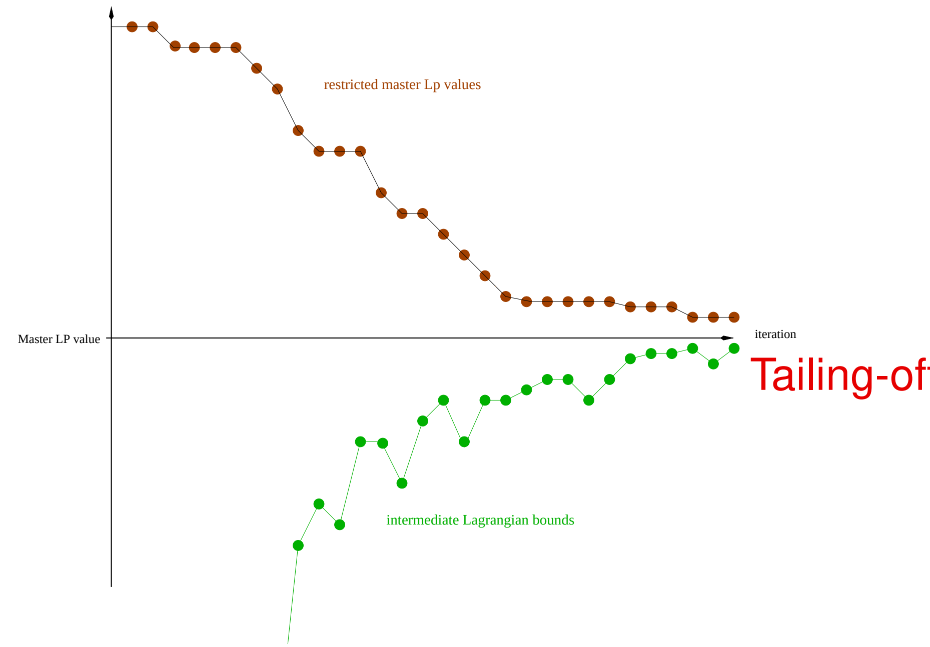


Instability: observed bad behaviors





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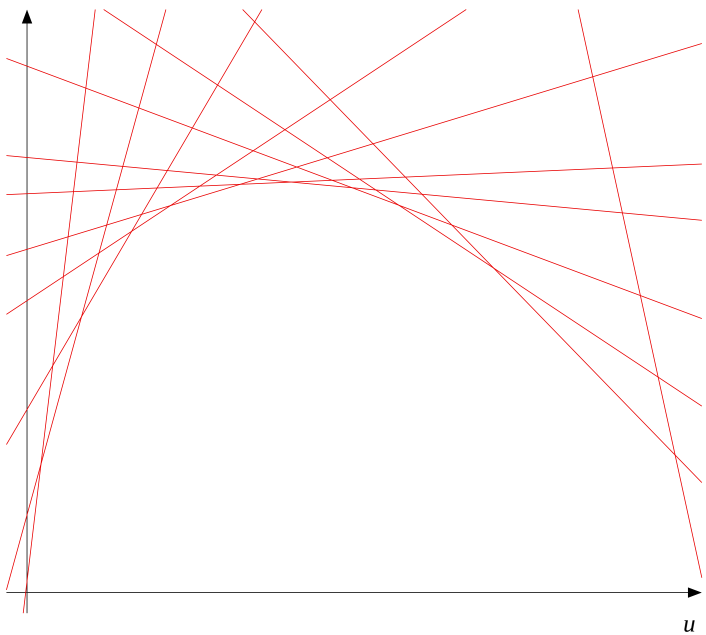

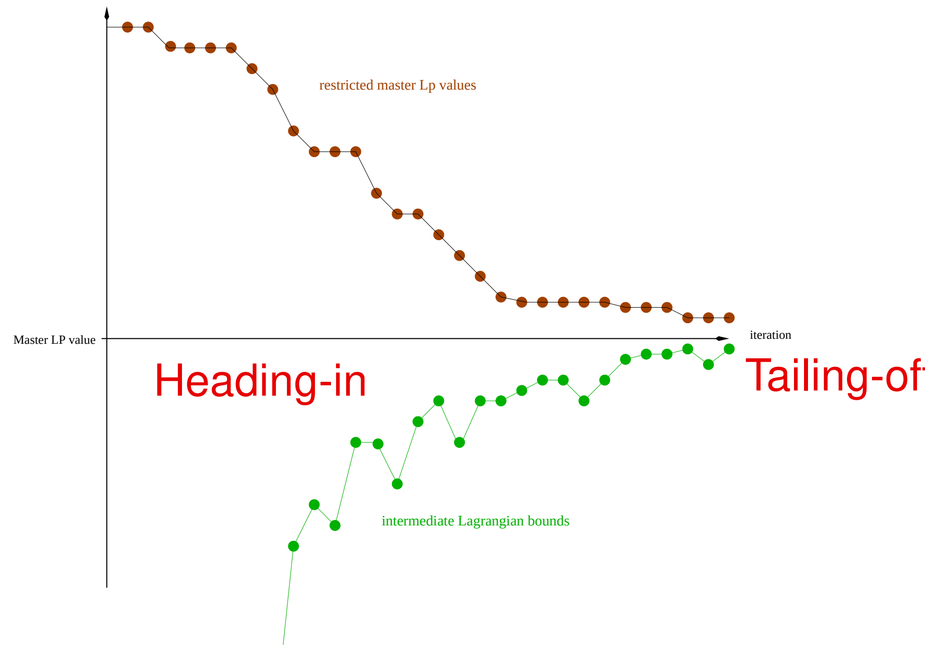

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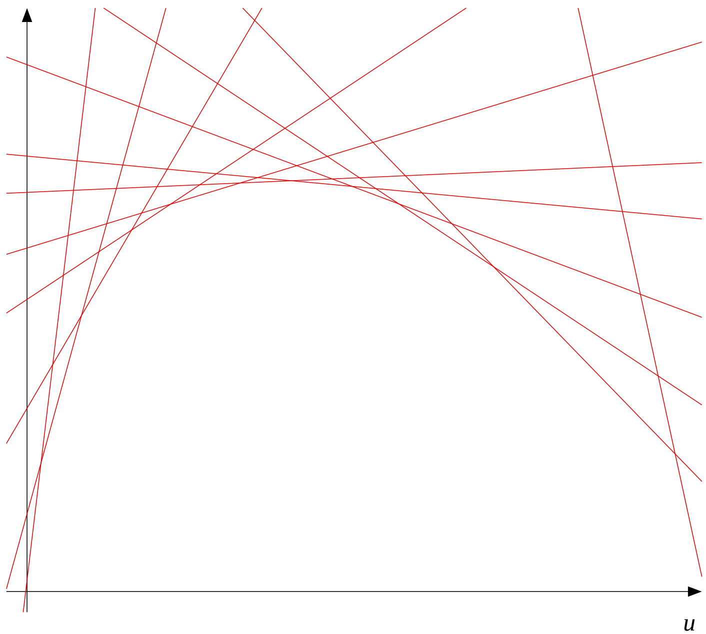

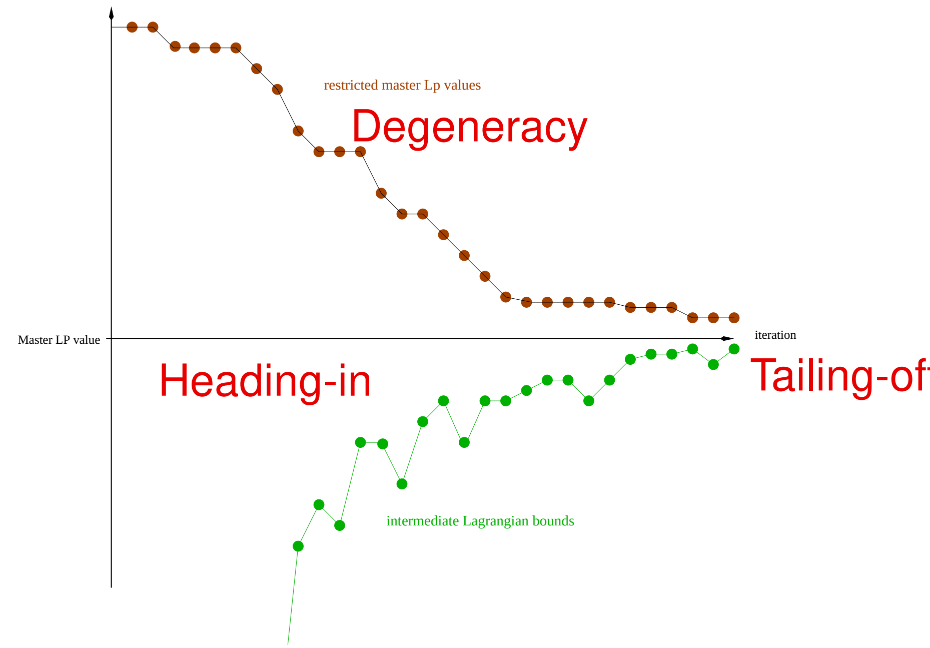

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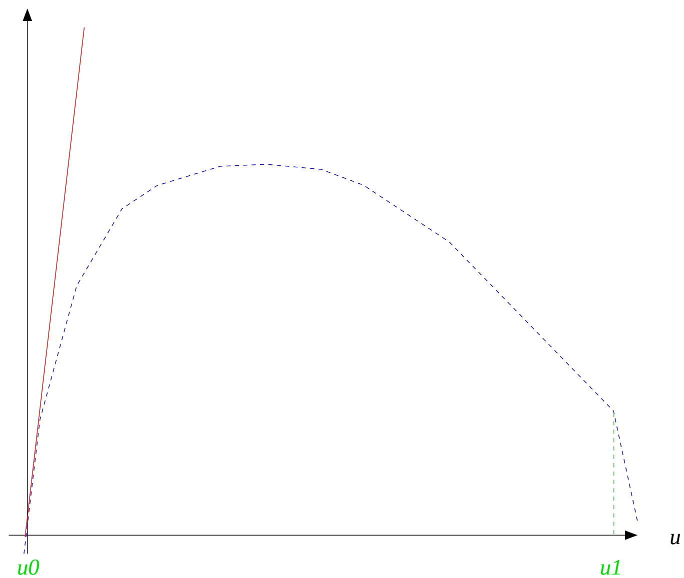
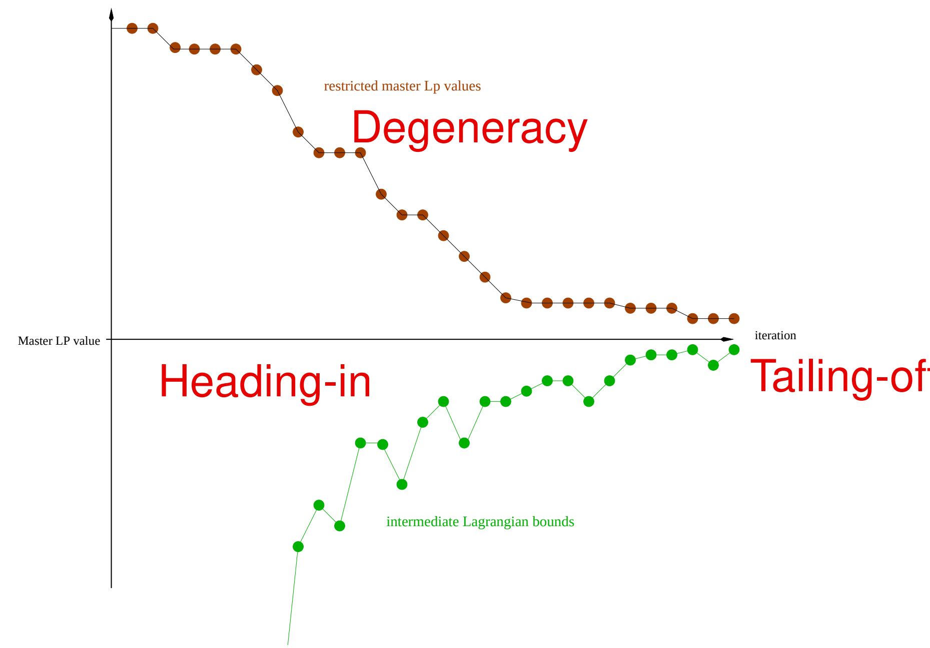


Bang-Bang Behavior



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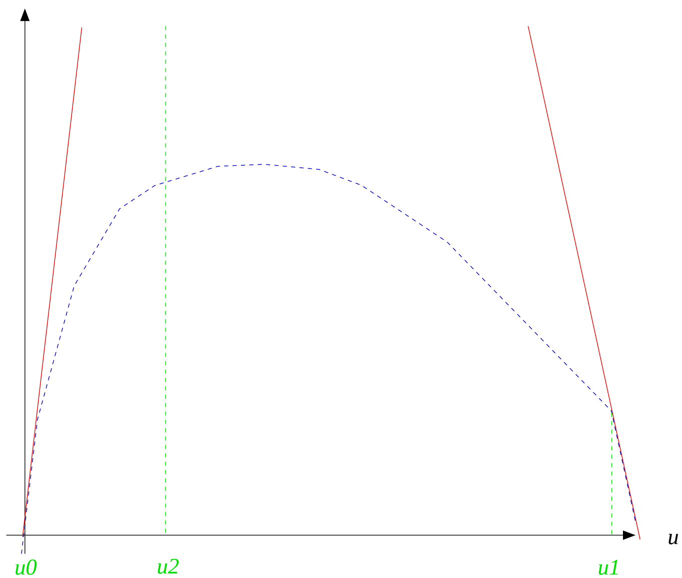

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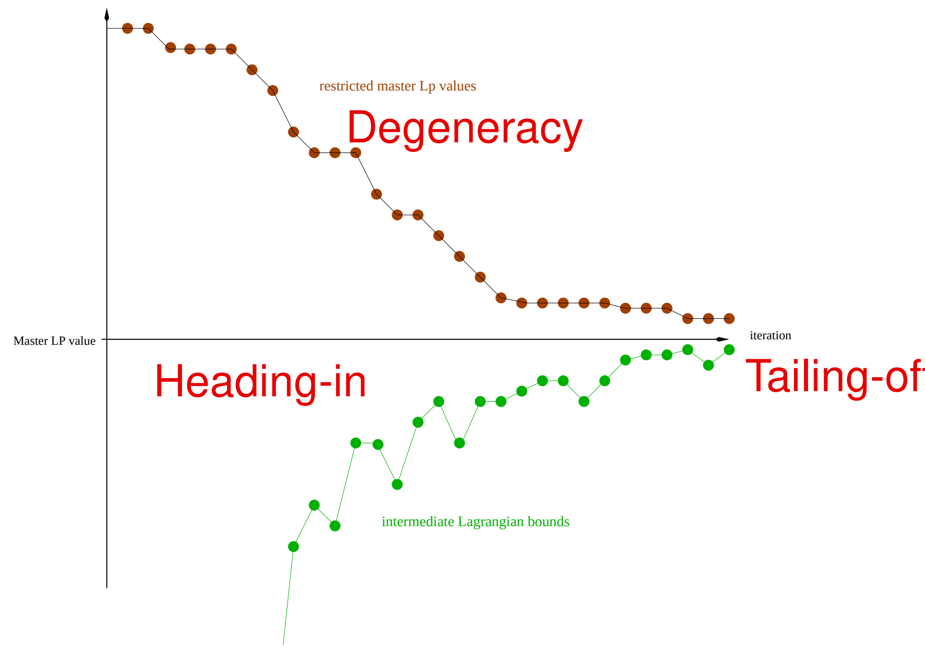

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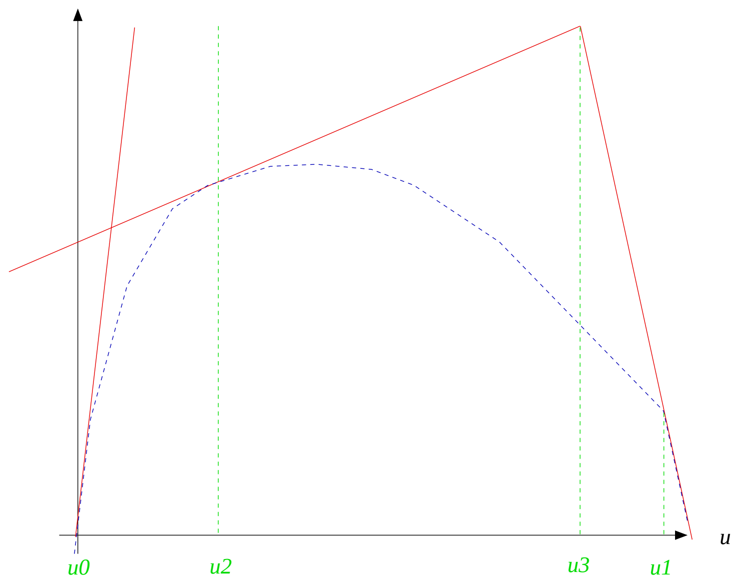

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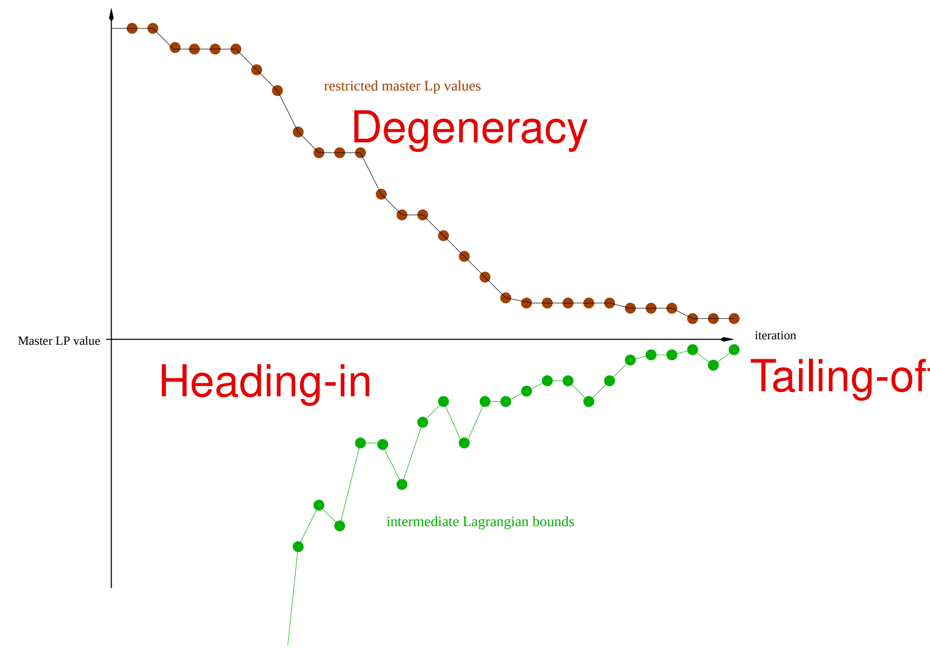

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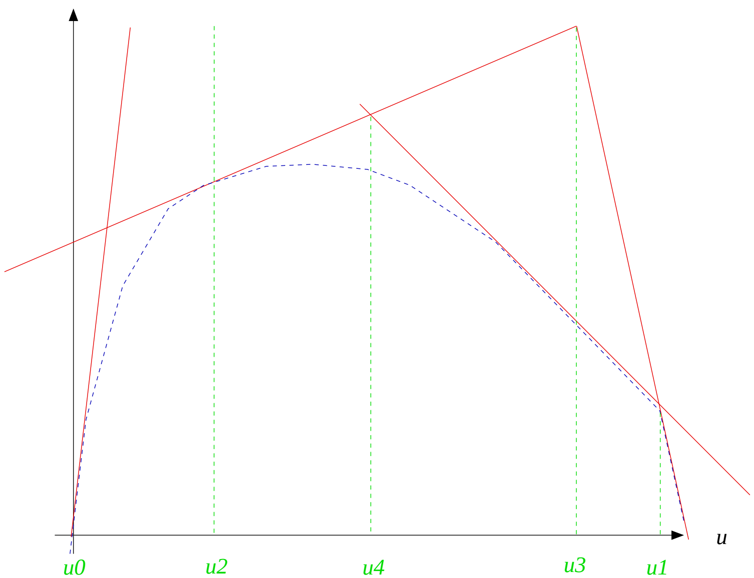

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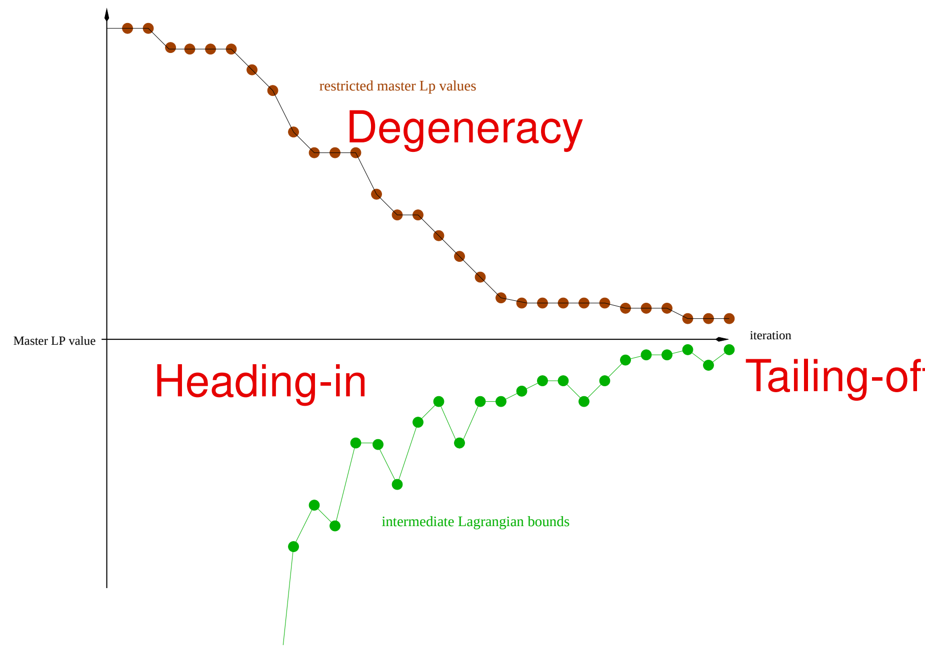

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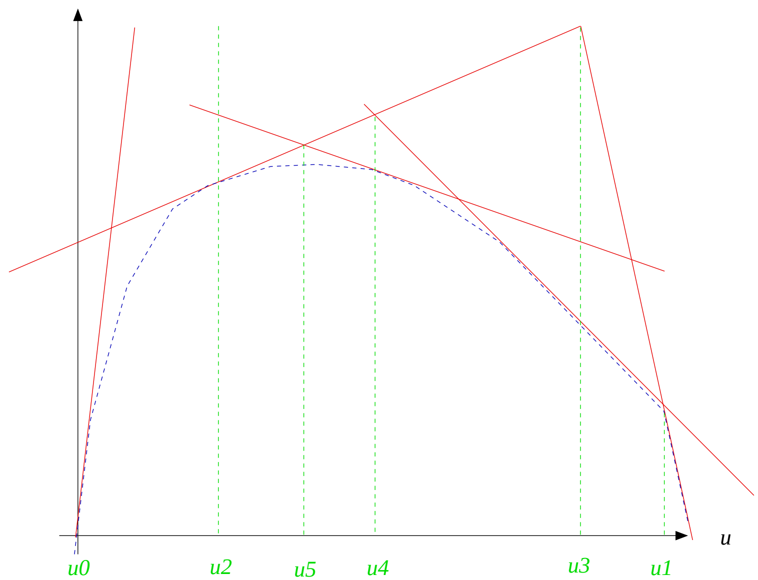

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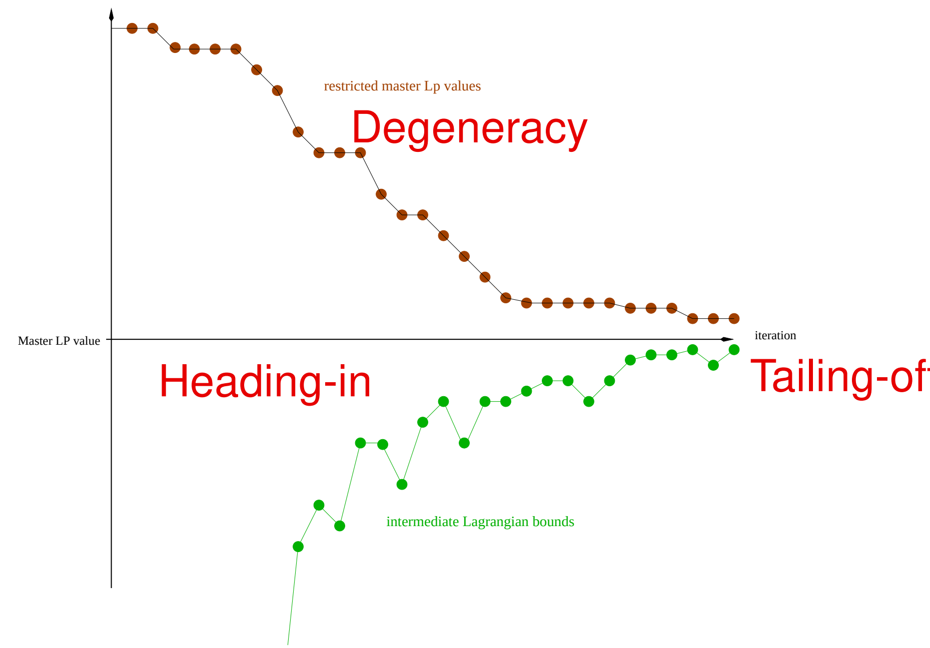

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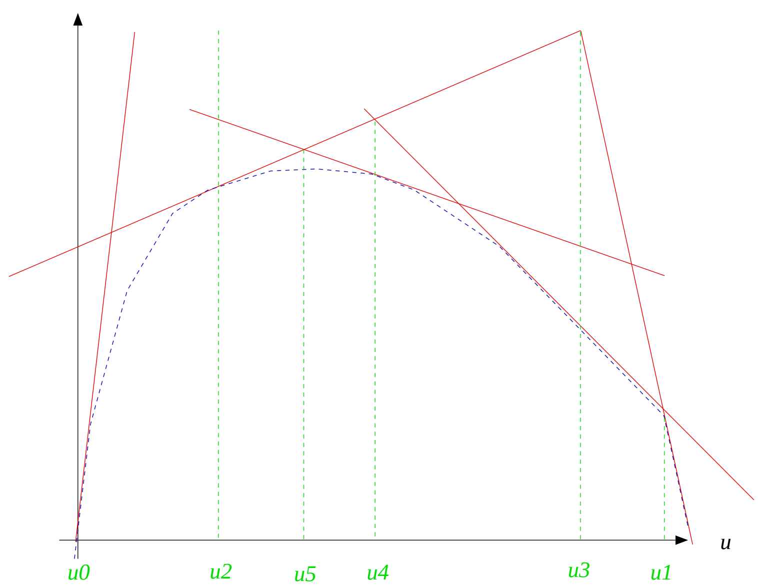

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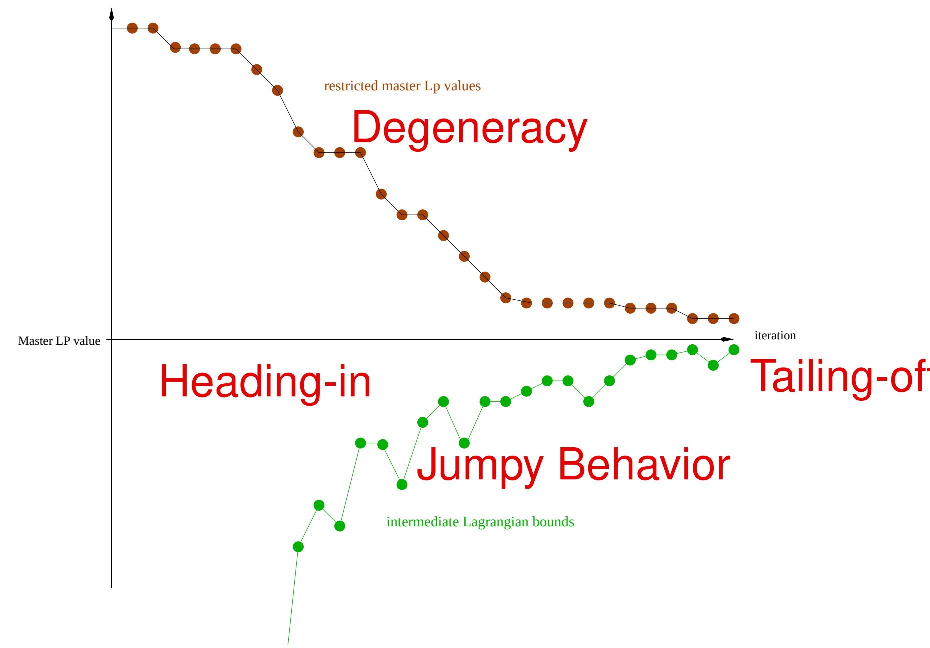

Stabilization Methods -

- penalizing distance from a stability center $\hat{u}$

$$\hat{u} \rightarrow \text{ best dual bound}$$

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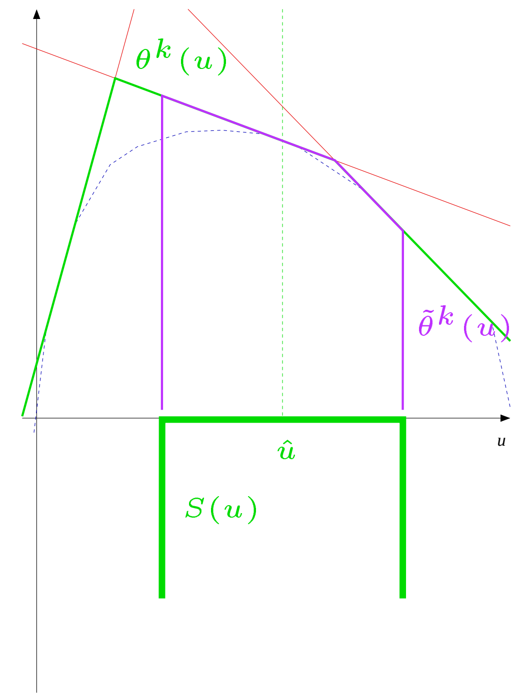
  R.E.Martsen, W.W.Hogan and J.W.Blankenship, 1975



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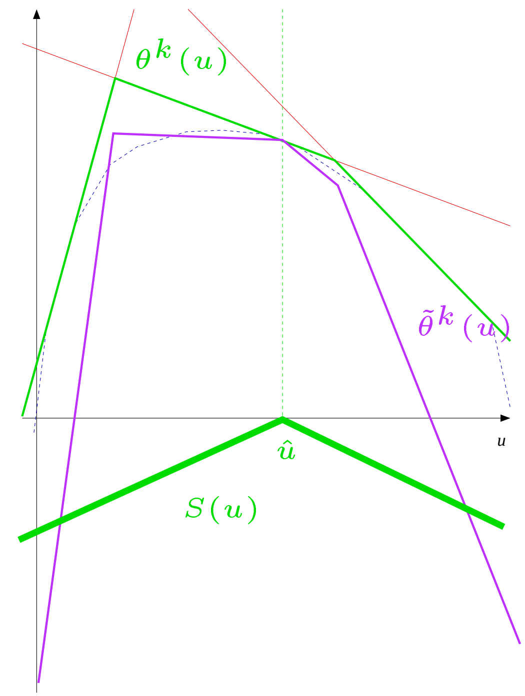
  $S(u - \hat{u}) = ||u - \hat{u}||_1$ (1 breakpoint)

  S.Kim, K.N.Chang and J.Y.Lee, 1995



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  $S(u - \hat{u}) = \delta(||u - \hat{u}||_\infty \leq \epsilon)$ (box step)

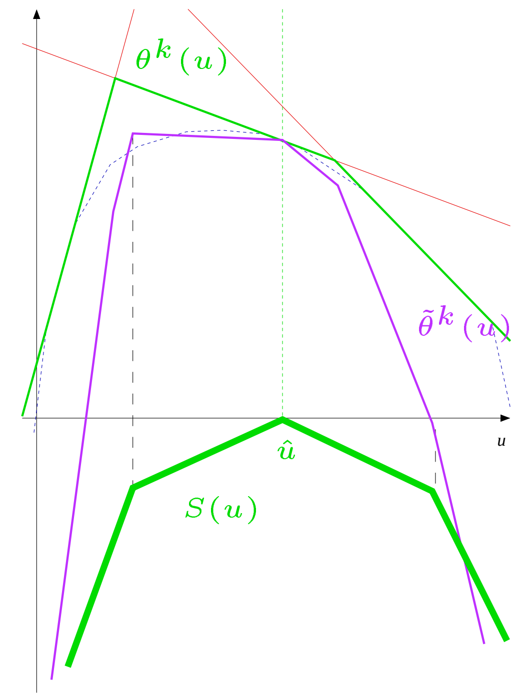
  R.E.Martsen, W.W.Hogan and J.W.Blankenship, 1975

  $S(u - \hat{u}) = ||u - \hat{u}||_1$ (1 breakpoint)

  S.Kim, K.N.Chang and J.Y.Lee, 1995

  $S(u - \hat{u}) = ||u - \hat{u}||_1$+penalization outside boxstep (3 breakpoints)

  O.du Merle, D.Villeneuve, J.Desrosiers and P.Hansen, 1999



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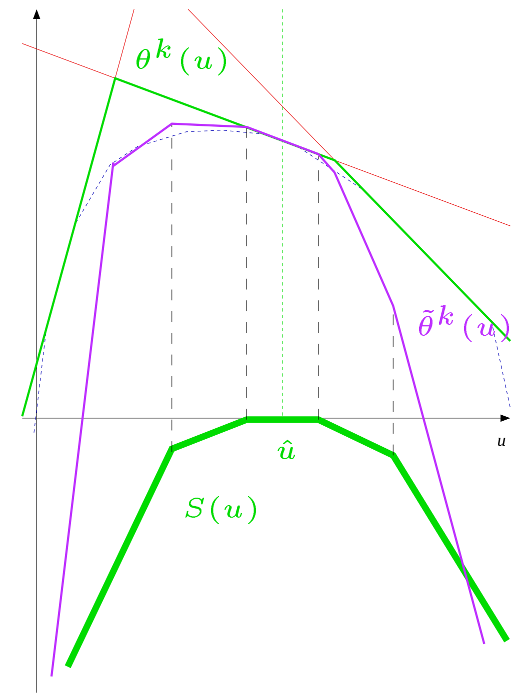
  $S(u-\hat{u})$ = penalization outside 2 boxsteps (4 breakpoints)

  H.Ben-Amor and J.Desrosiers, 2003



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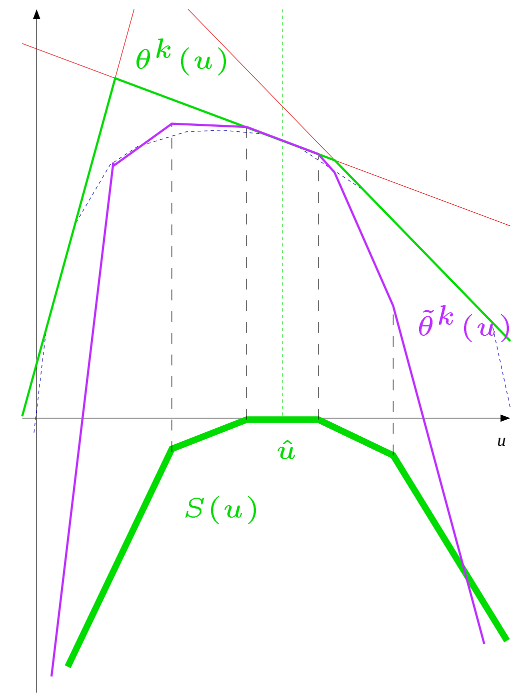

$\Rightarrow$ As $S(u - \hat{u})$ is a concave and piecewise linear function, we can still use LP solvers

Stabilization Methods - $\tilde{\theta}^k(u) = \theta^k(u) - S(u - \hat{u})$

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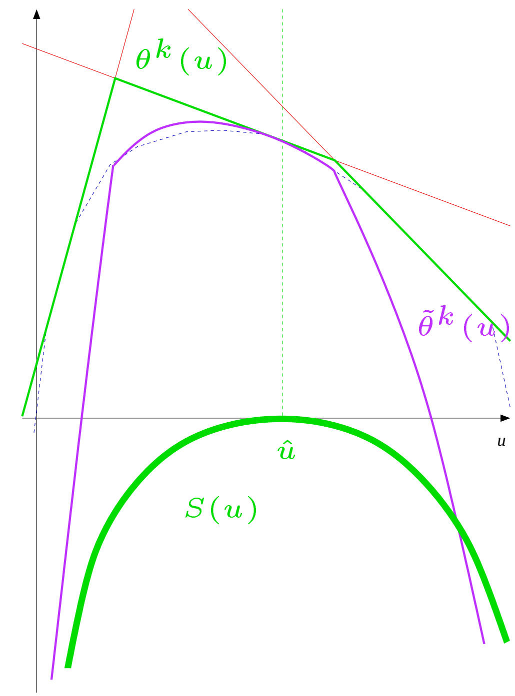
  H.Ben-Amor and J.Desrosiers, 2003

  $S(u - \hat{u}) = \frac{1}{2t}||u - \hat{u}||^2 \Rightarrow$ **Bundle**

  C. Lemaréchal and C. Sagastizábal, 1997



Stabilization Methods - $\tilde{\theta}^k(u) = \theta^k(u) - S(u - \hat{u})$

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  $S(u-\hat{u}) = \frac{1}{2t}||u-\hat{u}||^2 \Rightarrow$ **Bundle**

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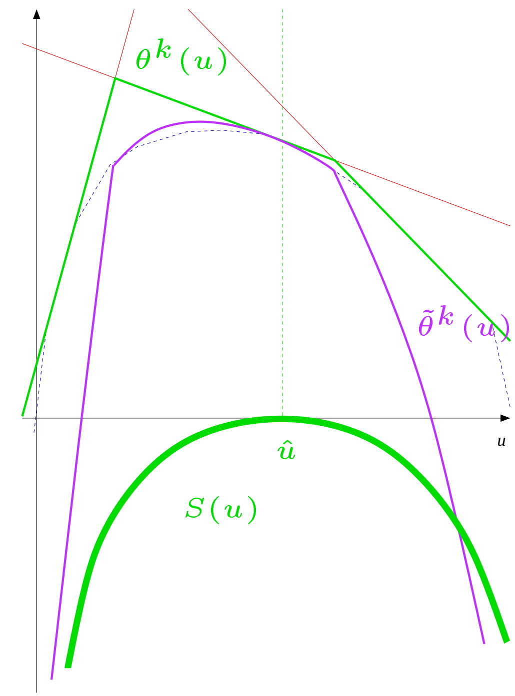

$\Rightarrow$ As $S(u-\hat{u})$ is a quadratic concave function,
we must now use Quadratic solvers *K.C.Kiwiel, 1989, 1994, A.Frangioni, 1996*

Other Stabilization Methods

- smoothing dual value : $u^k = \alpha\, u^{Kelley} + (1-\alpha)\, \hat{u}$

  P.Wentges, 1997

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- smoothing dual value : $u^k = \alpha\, u^{Kelley} + (1-\alpha)\,\hat{u}$

  P.Wentges, 1997

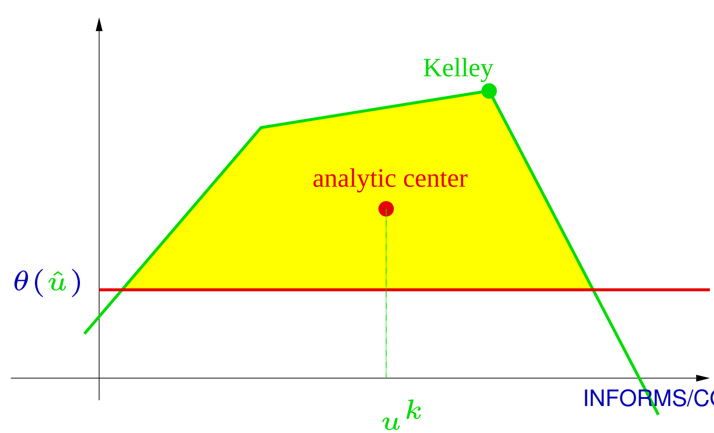
- Using interior-point-like technique to generate dual solution
  - Kelley solved by interior point method

    L-M Rousseau, M.Gendreau and D.Feillet, 2003

  - Analytic Center Cutting-Plane Method (ACCPM)

$$\tilde{\theta}^k(u) = \sum_{i \in I^k} \log(cx^i - u\,a\,x^i + r) + k\,\log(b\,u - r - \theta(\hat{u}))$$

J.L. Goffin, A.Haurie and J.Ph. Vial, 1992



Bi-Dual: Fenchel duality

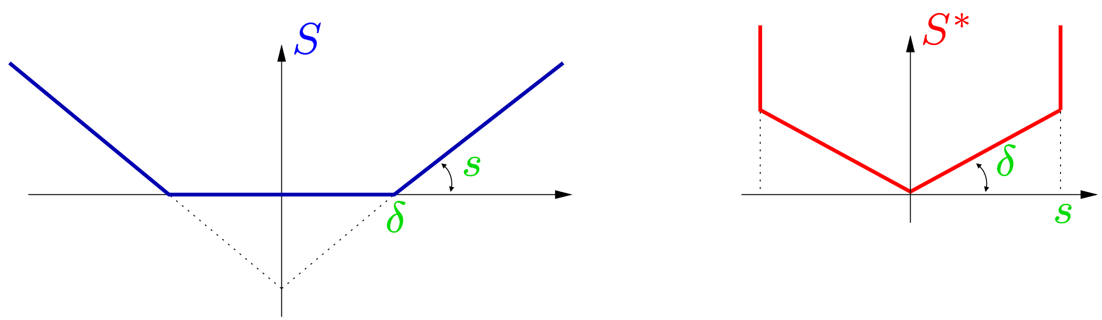
$$\begin{aligned} \max \theta^{k}(u) &\equiv \min\{c\,x : A\,x \geq b, x \in conv(X^{k})\} \\ \max \theta^{k}(u) - S(u - \hat{u}) &\equiv \quad \ldots \end{aligned}$$

Bi-Dual: Fenchel duality

$$
\begin{aligned}
\max \theta^{k}(u) \quad &\equiv \quad \min\{c\,x : \; A\,x \; \geq b, x \in conv(X^{k})\} \\
\max \theta^{k}(u) - S(u - \hat{u}) \quad &\equiv \quad \min\{c\,x + \hat{u}\; g + S^{*}(g) : \\
& \qquad A\,x \; + g \geq b, \; x \in conv(X^{k}), g \in \mathbb{R}^{m}\}
\end{aligned}
$$

Bi-Dual: Fenchel duality

$$
\begin{aligned}
\max \theta^k(u) \quad &\equiv \quad \min\{c\,x : \; A\,x \; \geq b, x \in conv(X^k)\} \\
\max \theta^k(u) - S(u - \hat{u}) \quad &\equiv \quad \min\{c\,x + \hat{u}\,g + S^*(g) : \\
& \qquad A\,x \; + g \geq b, \; x \in conv(X^k), g \in \mathbb{R}^m\}
\end{aligned}
$$



$$
\min \sum_{i=1}^{k} cx^i \lambda_i + \sum_{j=1}^{m} (\delta + \hat{u}_j) g_j^+ + \sum_{j=1}^{m} (\delta - \hat{u}_j) g_j^-
$$

$$
\begin{aligned}
\sum_{i=1}^{k} Ax^i \lambda_i + g_j^+ - g_j^- \quad &\geqslant \quad b_j \quad j = 1, \ldots, m\,, \\
s \geqslant g_j^+, \; g_j^- \quad &\geqslant \quad 0 \quad j = 1, \ldots, m\,, \\
\sum_{i=1}^{k} \lambda_i = 1, \;\; \lambda_i \quad &\geqslant \quad 0 \quad i\,.
\end{aligned}
$$

PART 2: Numerical Comparison

BaPCod : a generic Branch-And-Price Code

- $\exists$ '*tool-box*" codes : MINTO, ABACUS, BCP, or MAESTRO
- Our aim is to develop a '*black-box*" implementation (in the same way as Cplex or Xpress provide implementation of Branch-and-Bound)

BaPCod : a generic Branch-And-Price Code

- $\exists$ '*tool-box*" codes : MINTO, ABACUS, BCP, or MAESTRO
- Our aim is to develop a '*black-box*" implementation (in the same way as Cplex or Xpress provide implementation of Branch-and-Bound)
  - The user provides a MIP formulation of its problem
  - The user specifies what constraints make a subproblem
  - The code reformulates the problem for column generation
  - The code solves it with a default branch-and-price implementation

BaPCod : motivations

Practical:

- Provides a starting point to try out a column generation approach.
- Allows to test different decompositions: just change the subproblem definition.

Research Wise:

- Demands to generalize ad-hoc features to turn them into generic tools: initialization, preprocessing, branching, primal heuristic . . .
- Any algorithmic development is made available for all applications
- Permits to test algorithmic features across applications

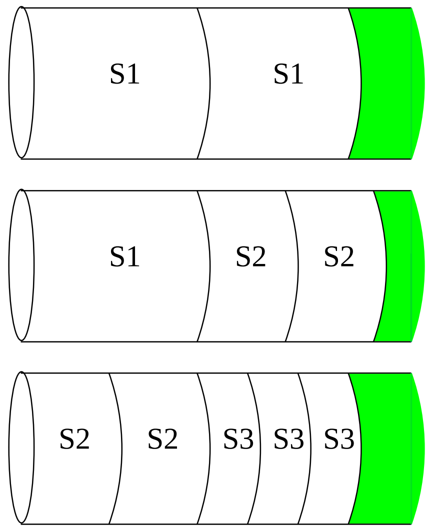
Tested variants

1. Kelley “Basic” initialized with 1 artificial column

2. Kelley “Rich” initialized with m artificial columns of cost $\alpha\hat{u}$ (with an heuristic $\hat{u}$)

3. Bundle initialized with $u_0 = \hat{u}$

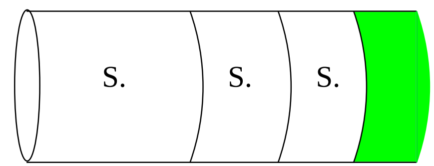
Cutting Stock Problem (CSP)



$\equiv$

$\sum_k x_{ik} \geq d_i \quad \forall i$

$+$



- $\min$ **number of rolls**

$$\begin{cases} \min & \sum_k y_k & & & \\ & \sum_k x_{ik} & \geq & d_i & \forall i \\ & \sum_i s_i x_{ik} & \leq & y_k & \forall k \end{cases}$$

- $\min$ **waste**

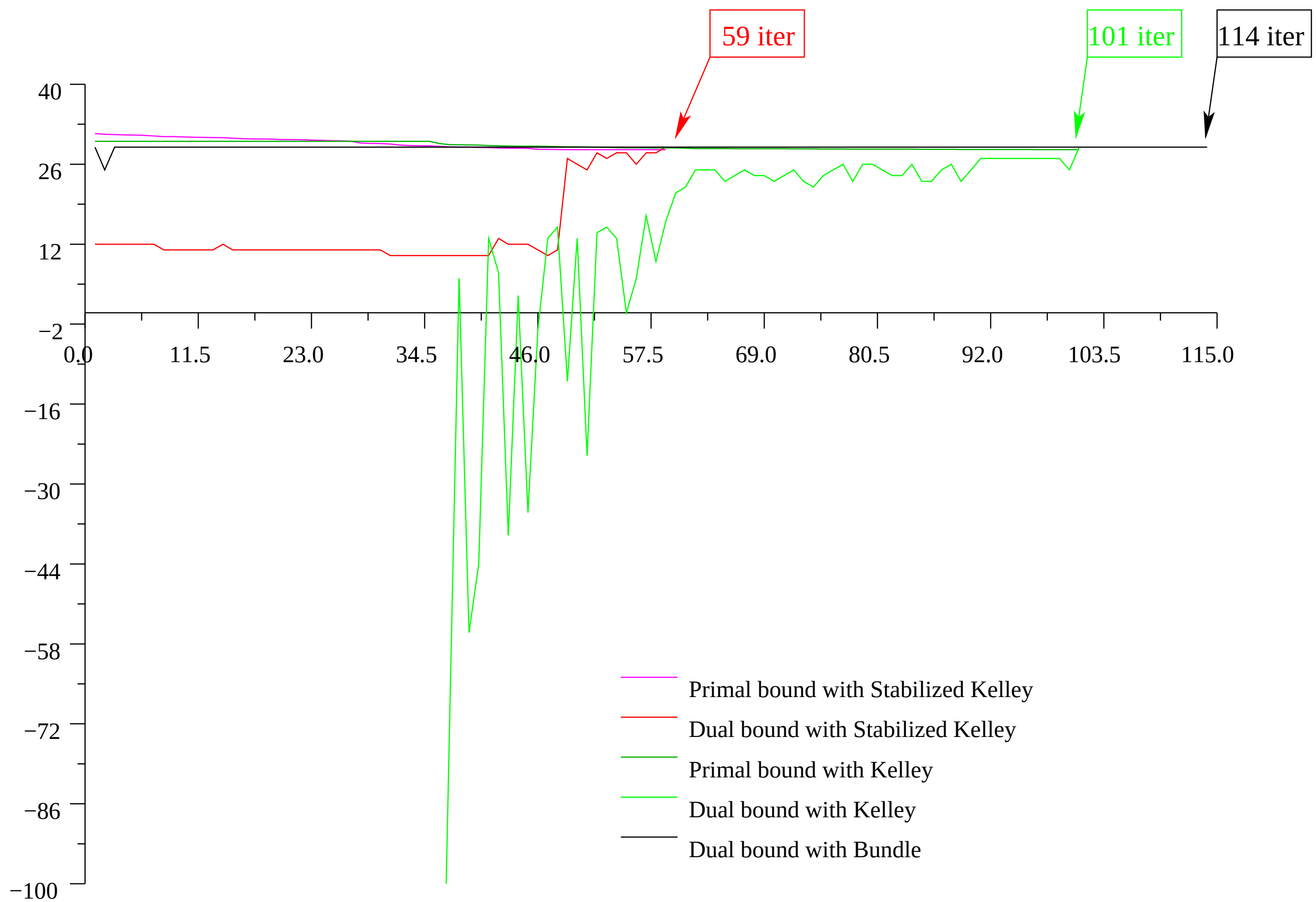
$$\begin{cases} \min & \sum_k (1 - \sum_i s_i x_{ik}) & & & \\ & \overline{d}_i \geq \sum_k x_{ik} & \geq & \underline{d}_i & \forall i \\ & \sum_i s_i x_{ik} & \leq & y_k & \forall k \end{cases}$$

CSP number of rolls: Comparative results

average on 10 or 20 instances

		Counters		Timers (ticks)		
Problem	Method	Iter	Col	Oracle	Master	total
realDat	kelley basic	45	46	54	2	0s64t
$i = 7 - 33$	kelley rich	29	29	422	2	4s30t
	bundle	51	38	1144	20	11s71t
randomDat	kelley basic	226	237	589	38	6s69t
$i = 50$	kelley rich	139	139	520	28	5s73t
	bundle	142	119	1703	260	19s89t

CSP number of rolls: primal and dual bounds

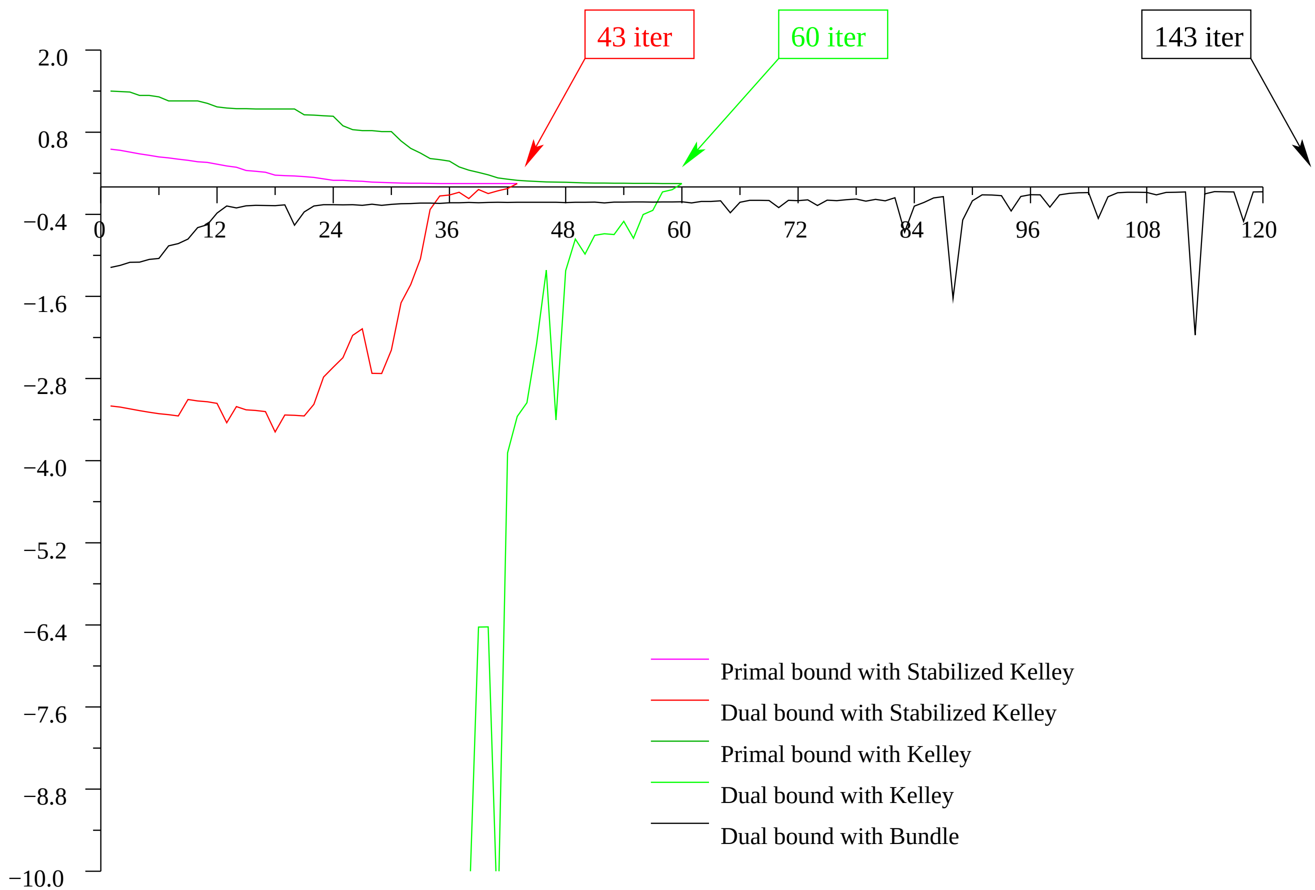


CSP Waste: Comparative results

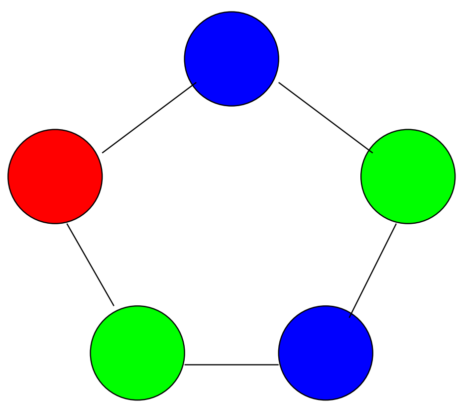
average on 10 or 20 instances

		Counters		Timers (ticks)		
Problem	Method	Iter	Col	Oracle	Master	total
realDat	kelley basic	43	43	239	3	2s53t
$i = 7 - 33$	kelley rich	31	26	842	3	8s53t
	bundle	60	35	1535	25	15s71t
randomDat	kelley basic	144	143	825	29	9s03t
$i = 50$	kelley rich	142	120	3280	29	33s56t
	bundle	200	86	5892	578	1m5s15t

CSP Waste: primal and dual bounds



Vertex Coloring problem (VCP)

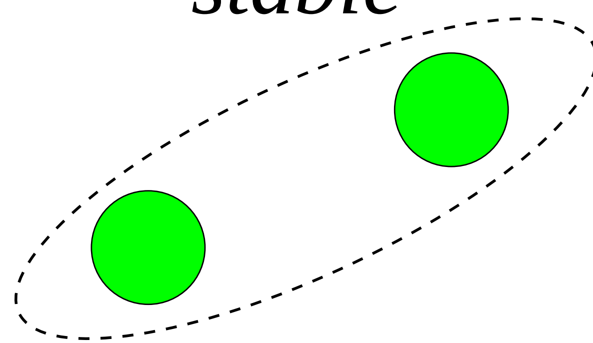


$\equiv$

$$\sum_k x_{ik} = 1 \quad \forall i$$

$+$

stable

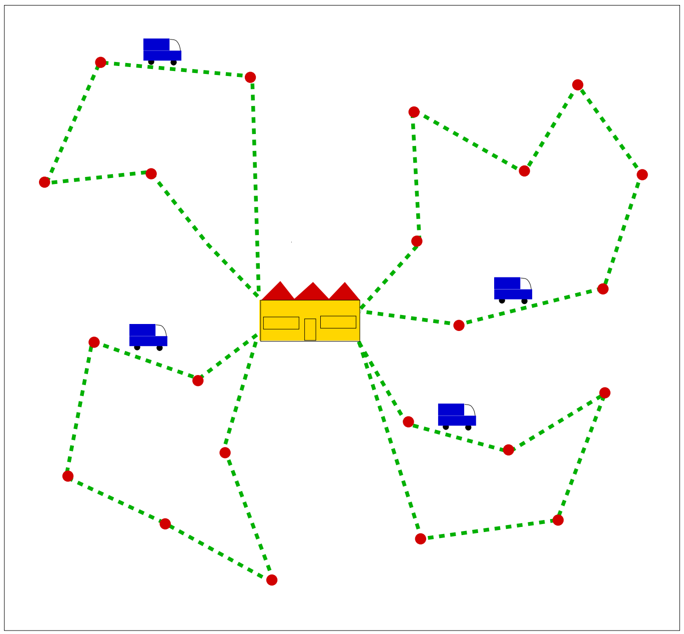


VCP : Comparative results

instances from DIMACS challenge
(mat.gsia.cmu.edu/COLOR/instances.html)

		Counters		Timers (ticks)		
Problem	Method	Iter	Col	Oracle	Master	total
Mycielski	kelley basic	94	94	6391	83	1m08s73t
$n = 5 - 47$	kelley rich	69	69	6612	66	1m11s54t
$m = 5 - 236$	bundle	41	22	3853	1949	1m01s90t
Queen	kelley basic	183	183	119174	267	20m32s12t
$n = 25 - 81$	kelley rich	128	128	88701	179	15m19s80t
$m = 160 - 2112$	bundle	50	46	13610	2380	15m22s31t

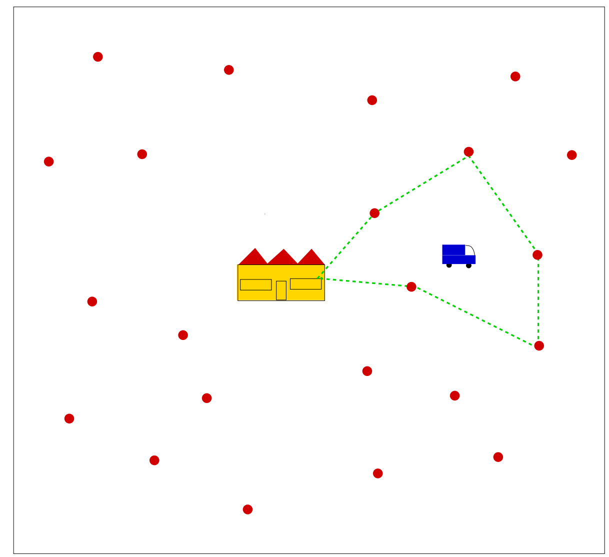
Capacitated Vehicle Routing Problem (CVRP)



$\equiv$

$$\sum_{k \in K} x_{ik} \geq 1 \quad \forall i \neq 0$$

$+$

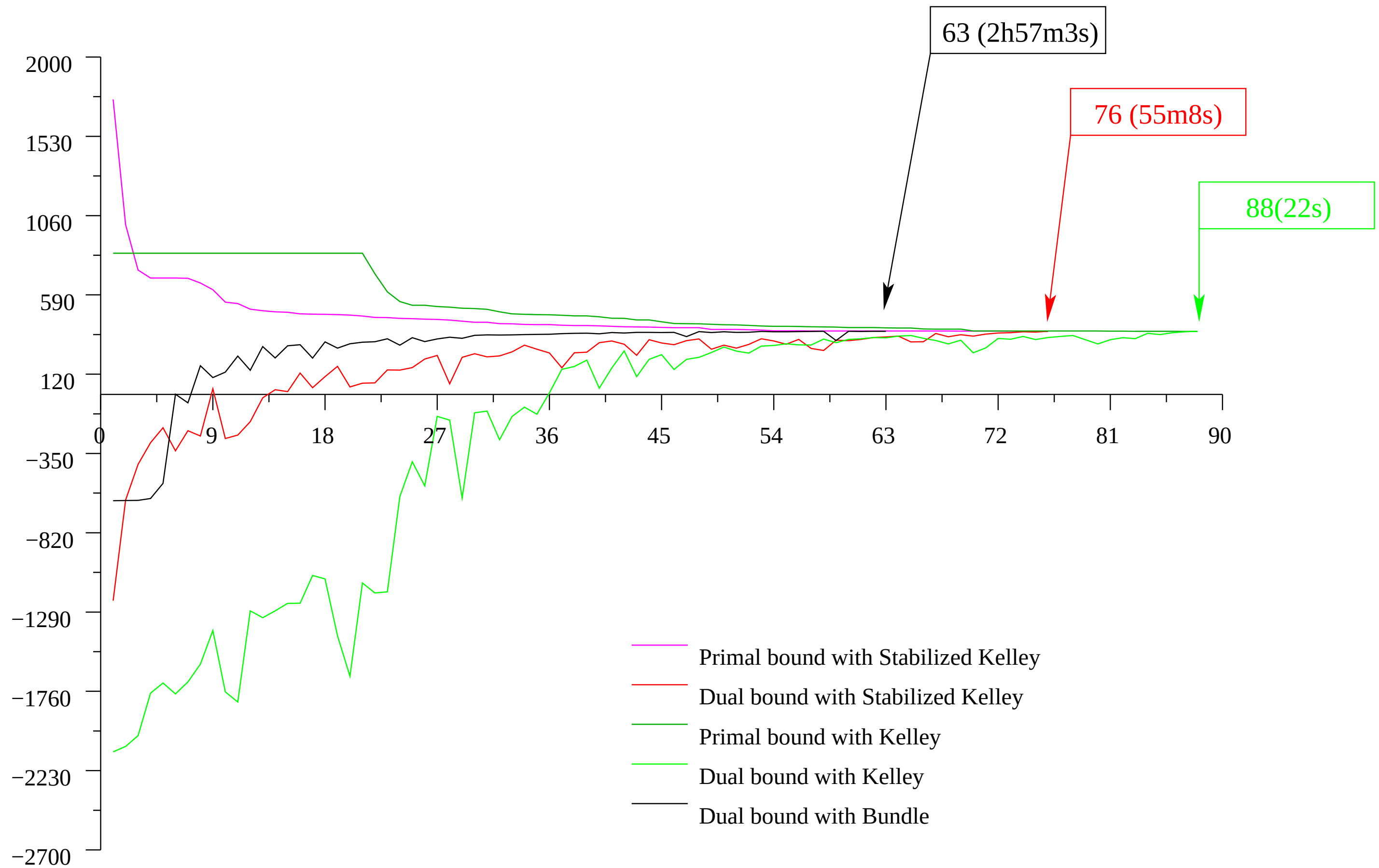


CVRP : Comparative results

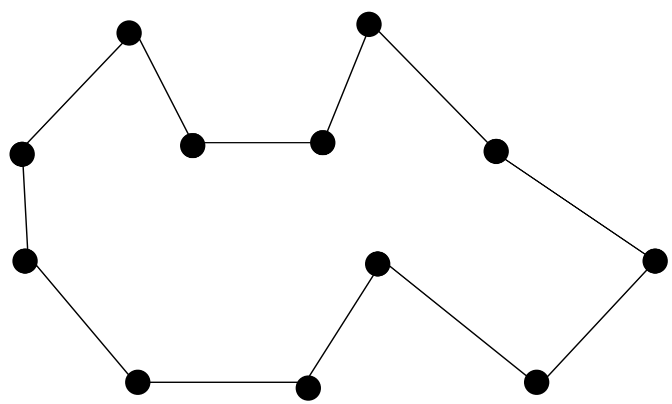
Instances and generated instances from CVRPLIB (www.branchandcut.org/VRP/data)

		Counters		Timers (ticks)		
Problem	Method	Iter	Col	Oracle	Master	total
P-k8	kelley basic	49	48	21	7	0s35t
average on 3 inst	kelley rich	42	40	22	11	0s43t
n $= 16 - 23$	bundle	59	32	76	33	1s21t
E-n22-k4	kelley basic	88	88	2144	24	21s84t
	kelley rich	76	75	330826	22	55m8s59t
	bundle	63	47	1062299	26	2h57m3s45t
gen-k2	kelley basic	34	35	4044	3	40s53t
average on 8 inst	kelley rich	28	28	3046	4	30s57t
n $= 8 - 14$	bundle	17	13	10490	6	1m45s0t

CVRP : E-n22-k4 : primal and dual bounds



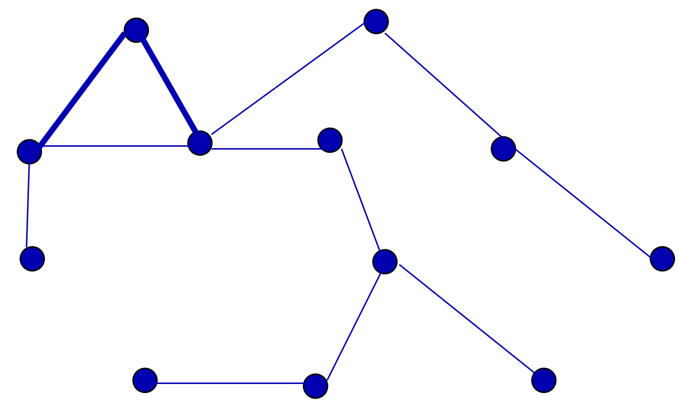
Traveling Salesman Problems (TSP)



$\equiv$

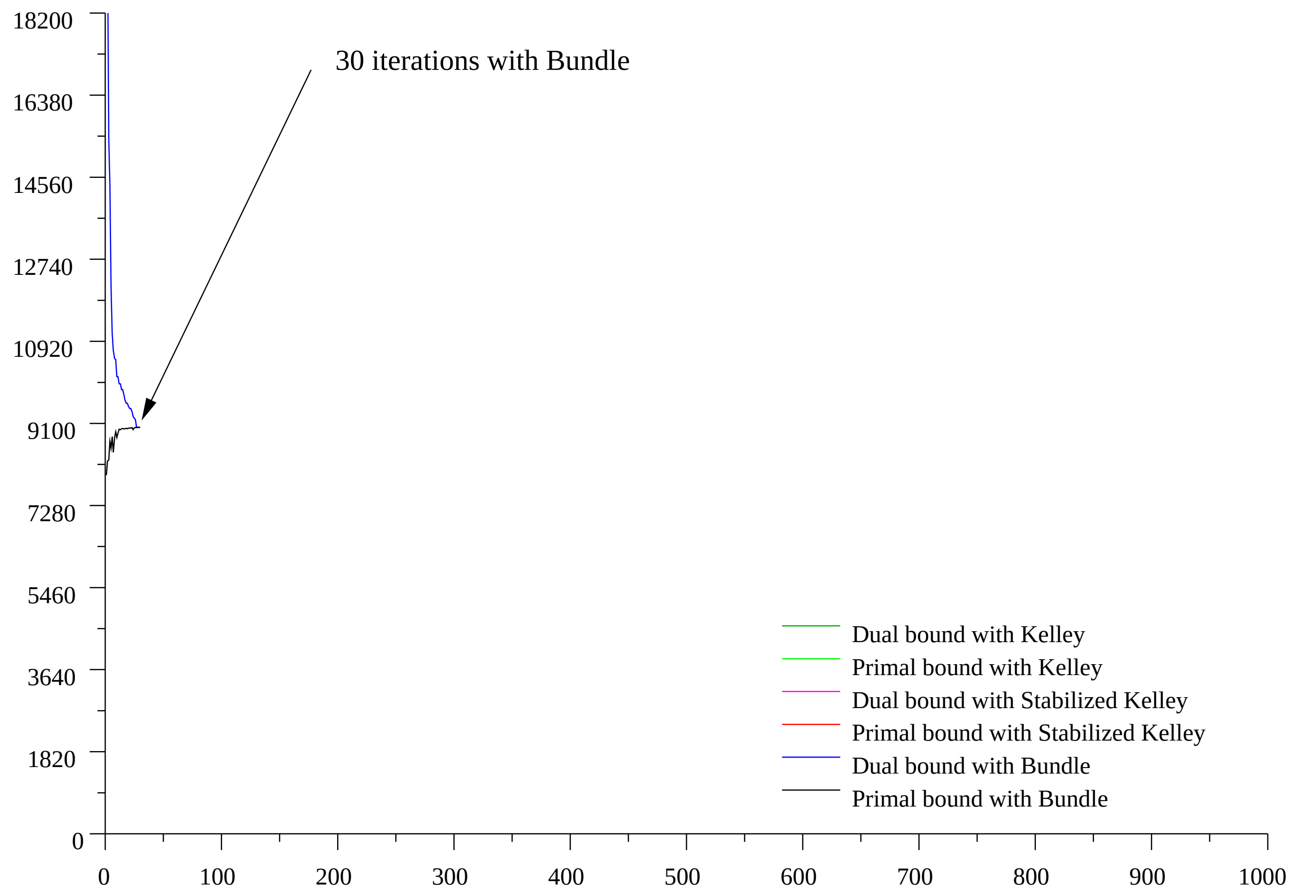
$\sum_{e \in \delta(i)} x_e = 2 \;\; \forall i$

$+$

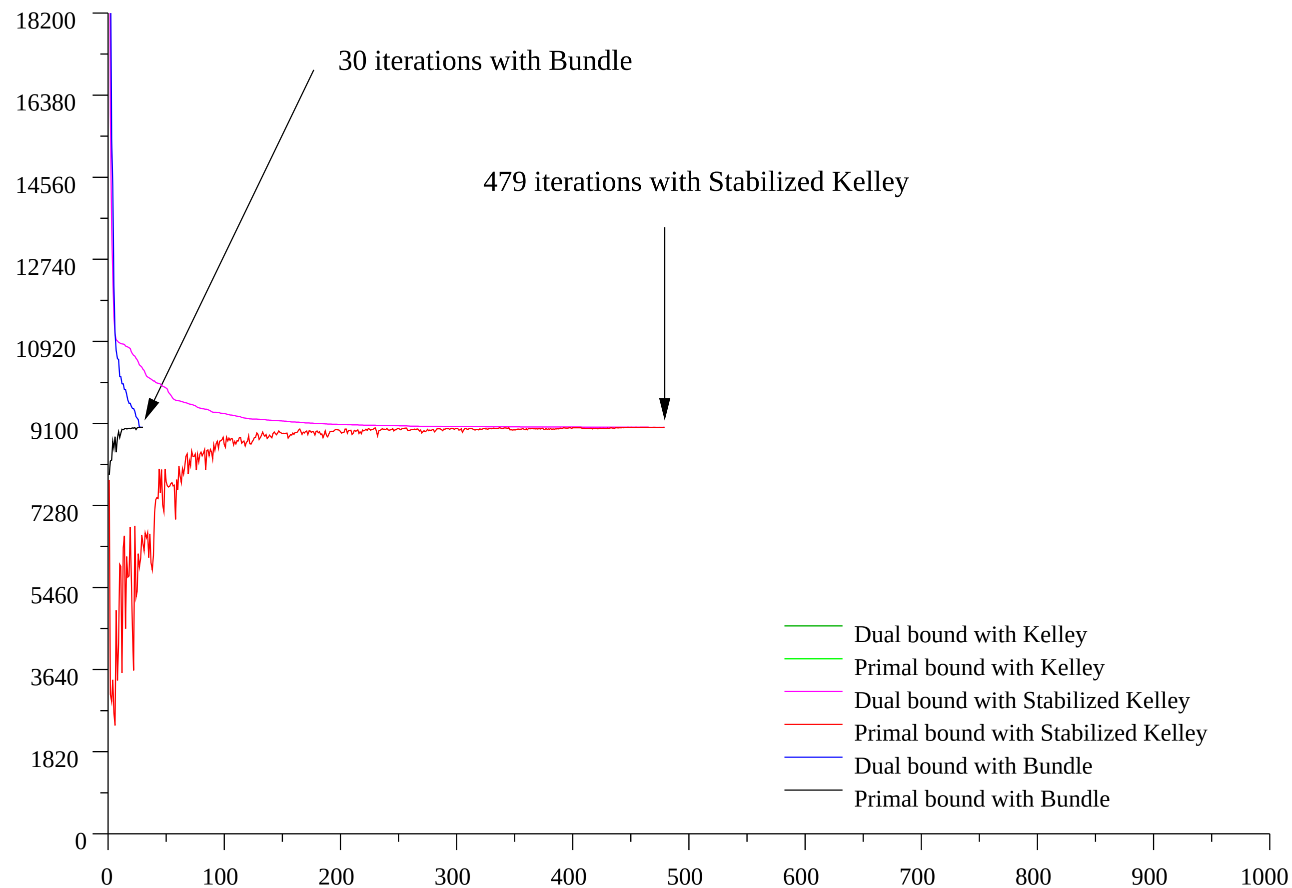


Instances from TSPLIB

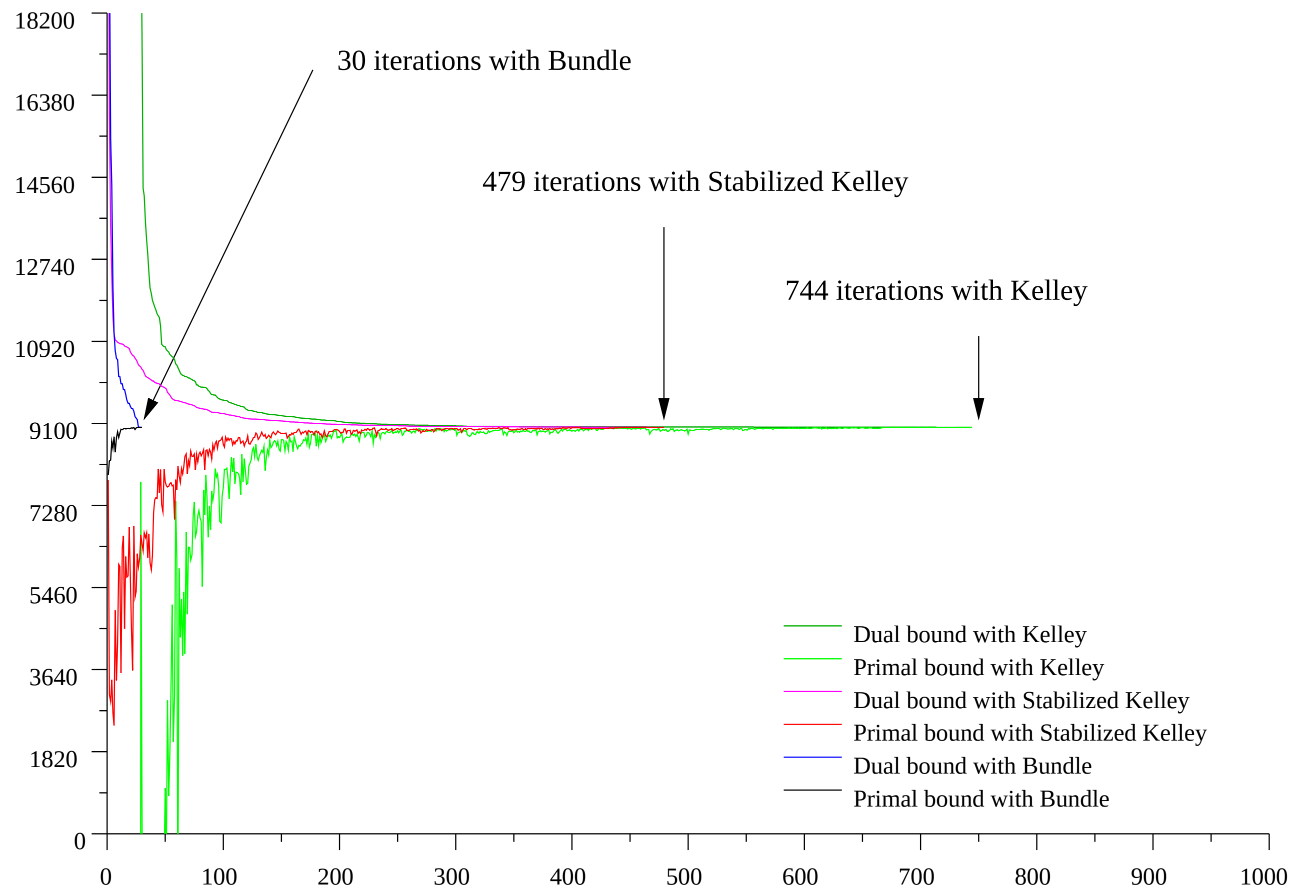
TSP : bays29 : primal and dual bounds



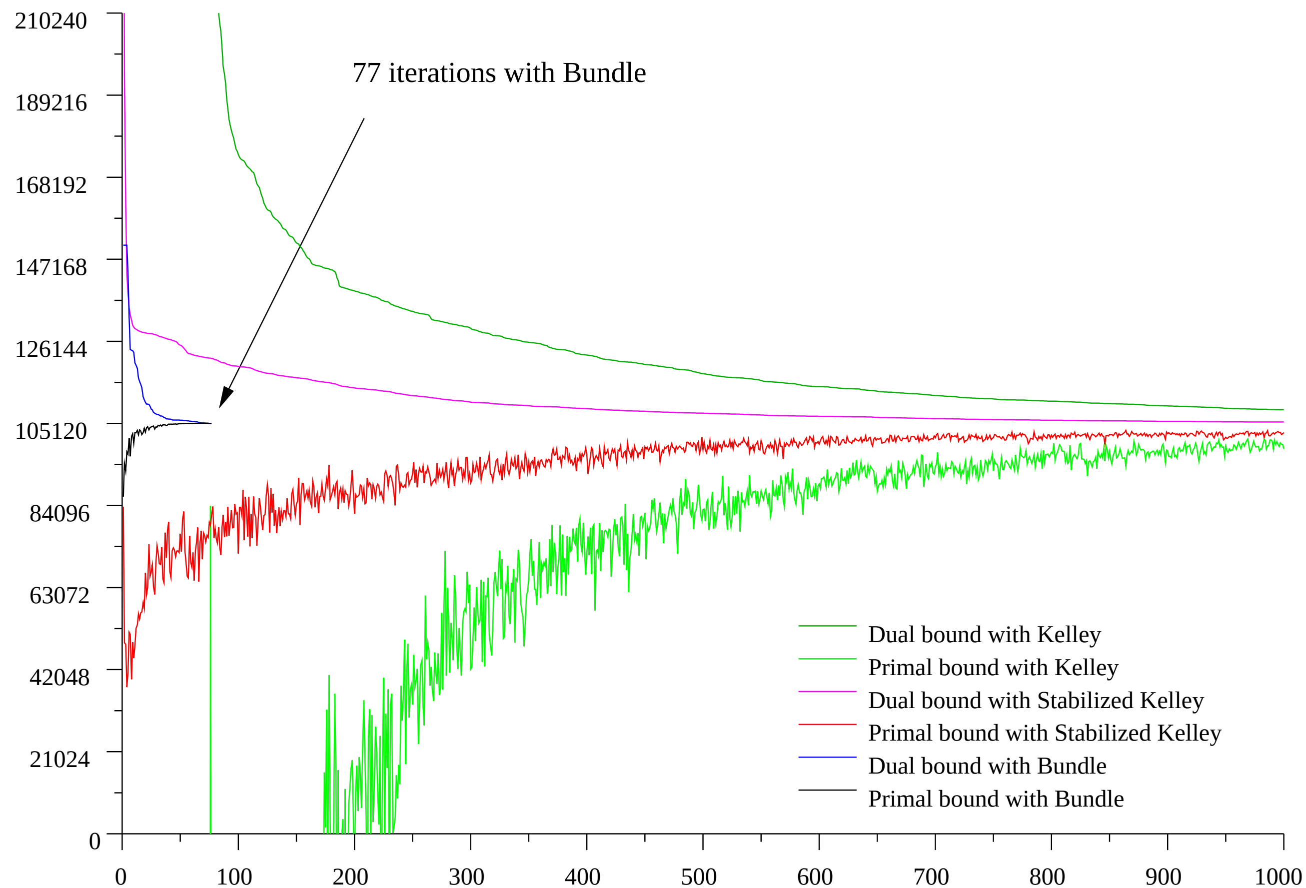
TSP : bays29 : primal and dual bounds



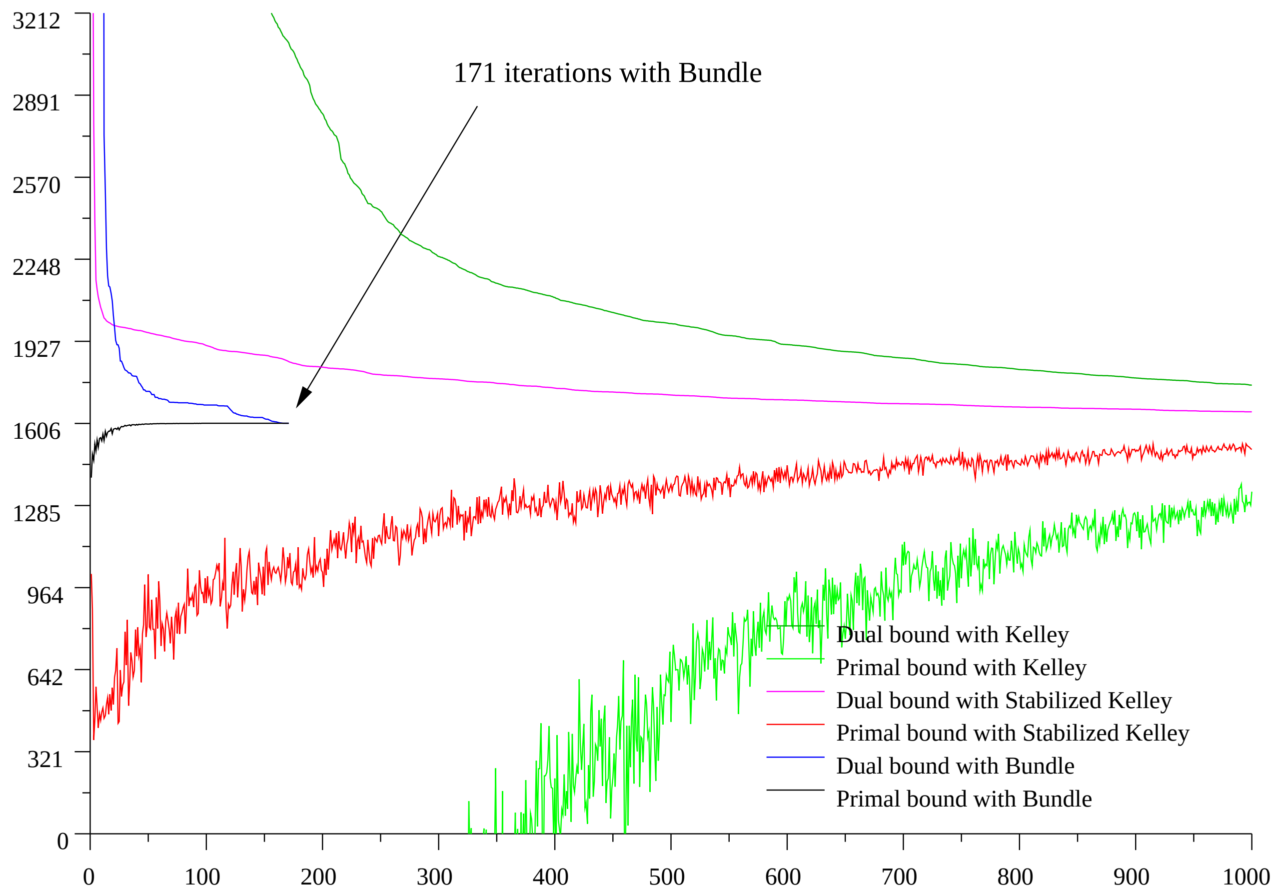
TSP : bays29 : primal and dual bounds



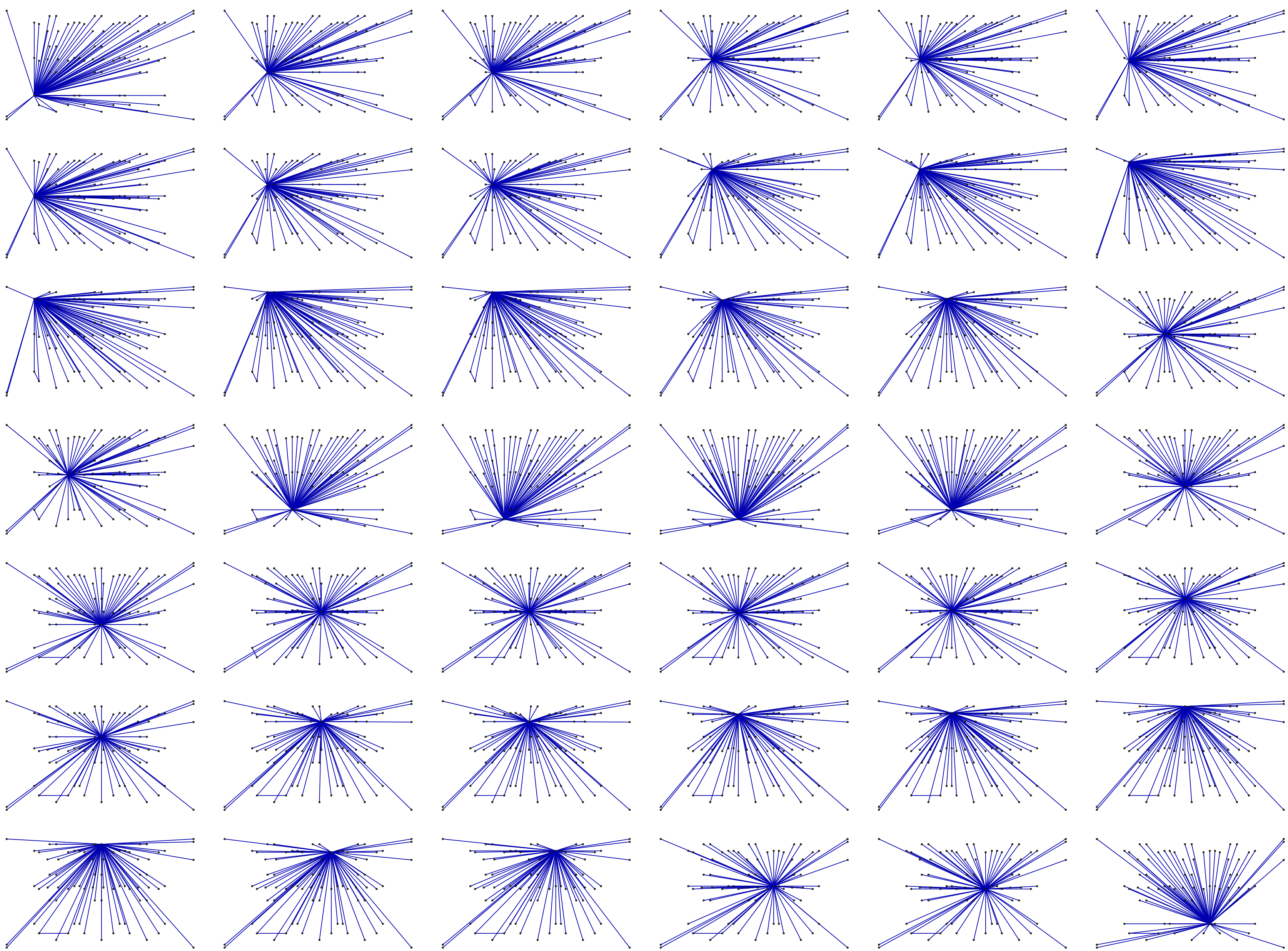
TSP : pr76 : primal and dual bounds



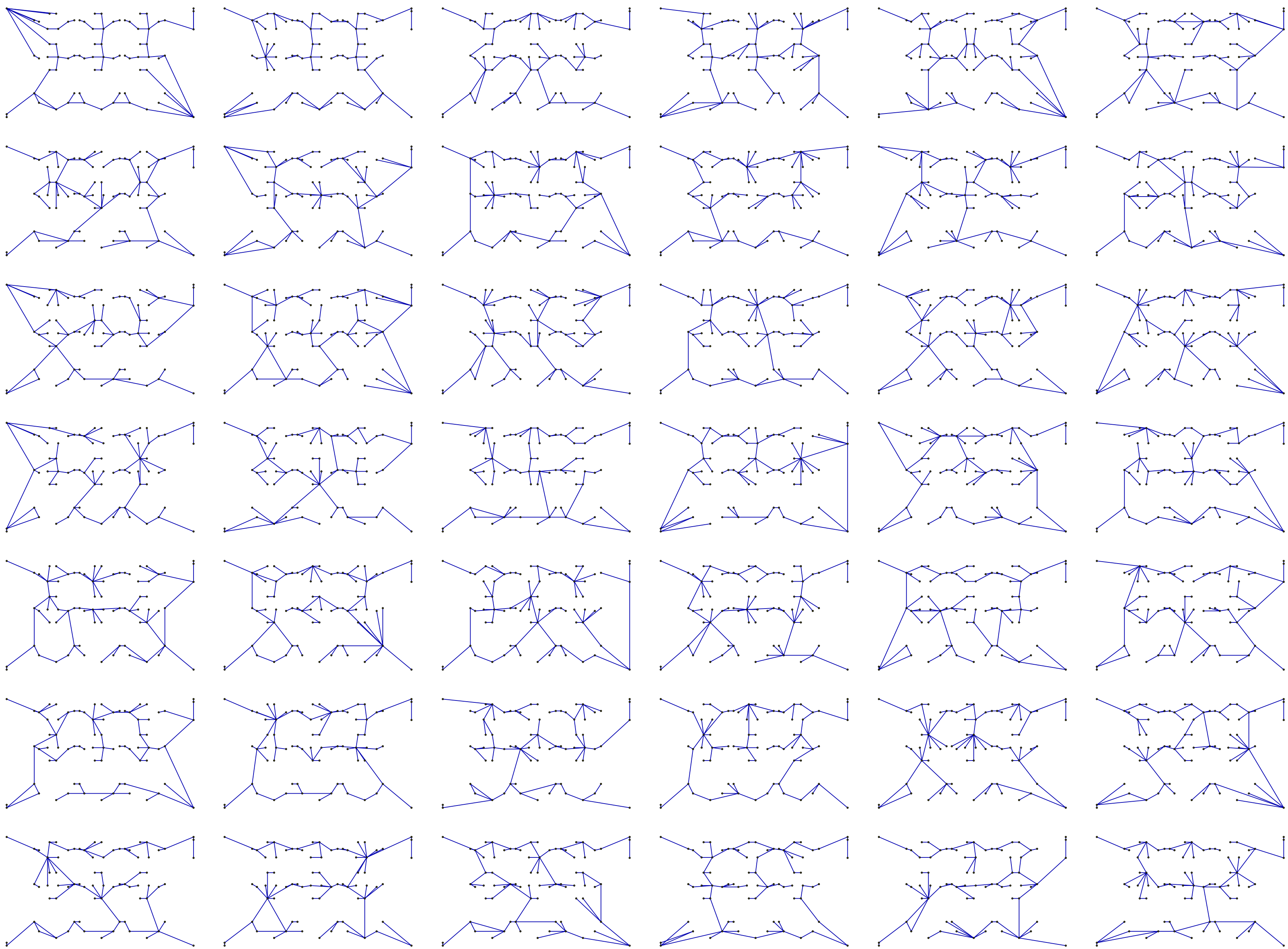
TSP : gr120 : primal and dual bounds



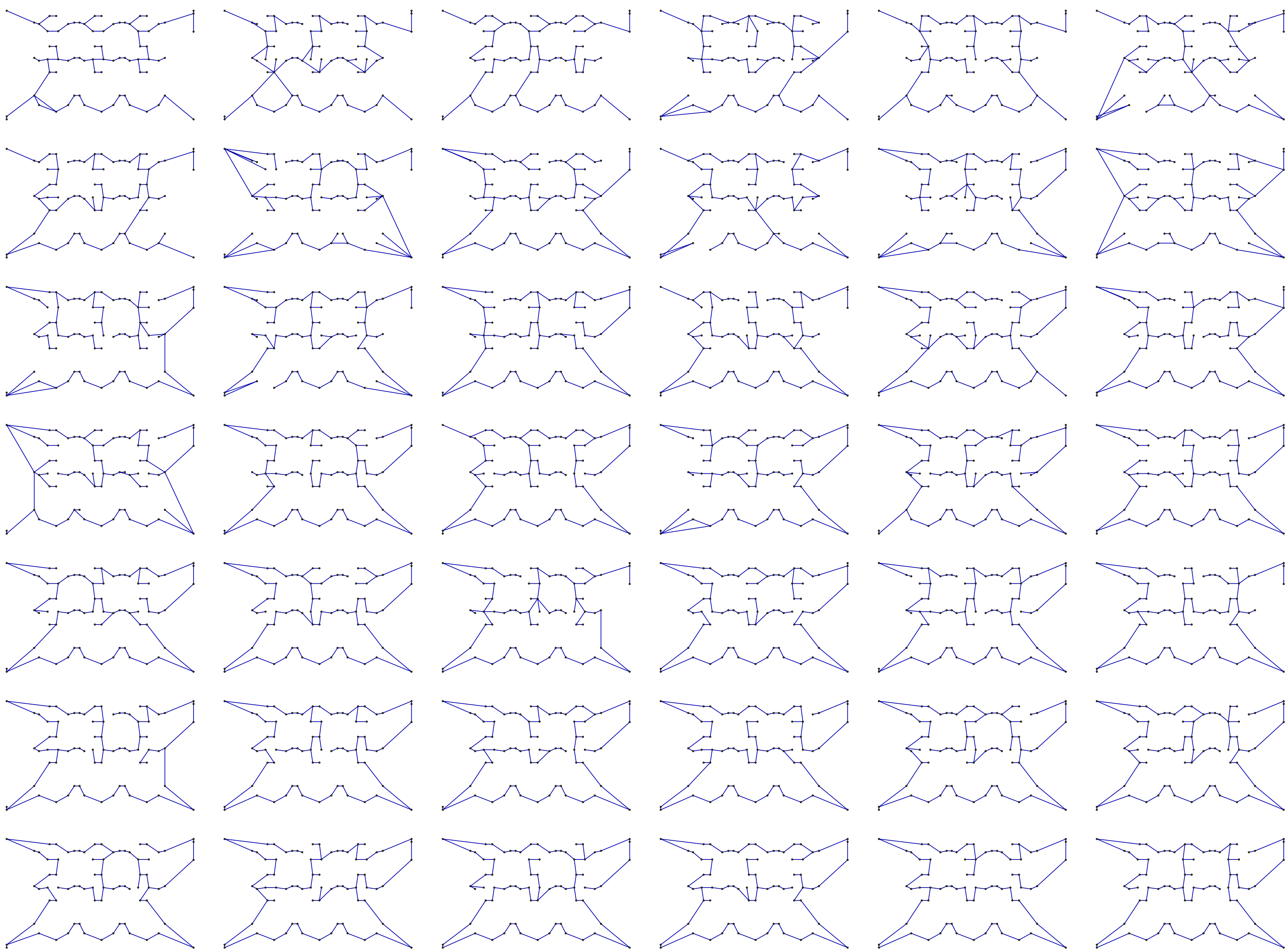
pr76 : first 1-trees with Kelley



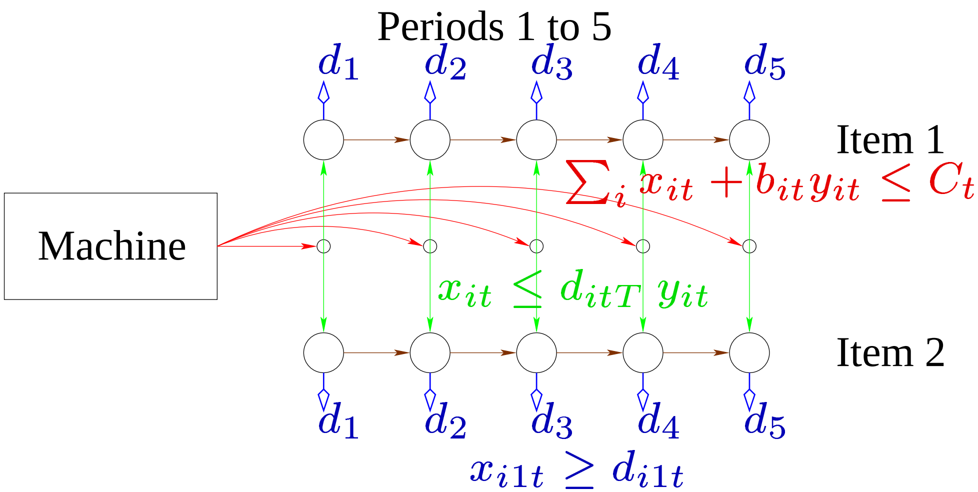
pr76 : first 1-trees with Stabilized Kelley



pr76 : first 1-trees with Bundle



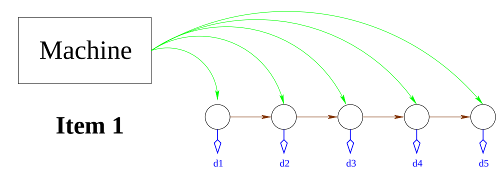
Multi-Item Lot-Sizing (MILS)



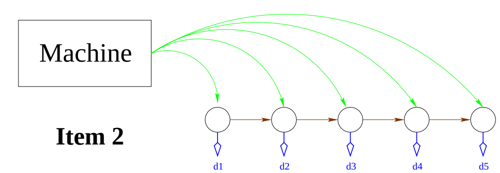
$$\equiv$$

$$\sum_i x_{it} + b_{it} y_{it} \leq C_t \quad \forall t$$

$$+$$



$$AND$$



MILS : Comparative results

average on 10 instances

		Counters			Timers (ticks)		
Problem	Method	Iter	SP	Col	Oracle	Master	total
i20-t60	kelley basic	9	164	117	60	3	5s08t
	kelley rich	15	244	80	86	7	4s45t
	bundle	34	672	75	326	610	15s73t

Observations

(based on preliminary results)

- Bundle takes far fewer iterations for some applications (like TSP), while not much is to win (compared to kelley) in other applications.

  ? Characterization ?

- Stabilization may imply harder oracles. Hence, there is a time tradeoff (warm start for the oracle).
- Kelley finds primal feasible solution earlier.
- Kelley accepts inexact oracles and multiple generation columns (to be implemented for Bundle).